ELEKTROSTATİKTE SINIR DEĞER PROBLEMLERİ I

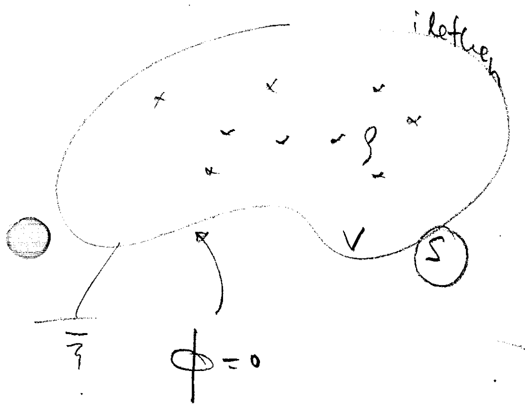
(i) Görüntü yükleri yöntemi

(ii) Dik fonksiyonlar cinsinden çözüm) bunları inceleyeceğiz

(iii) kompleks değişkenler yöntemi

2.1. Görüntü yükleri yöntemi

Yük dağılımı olarak noktasal yükler ve iletken sınır yüzeyler verilmişse bu yöntem uygulanır.

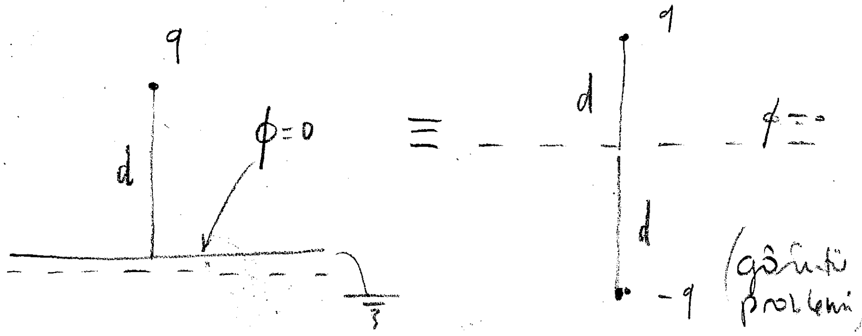


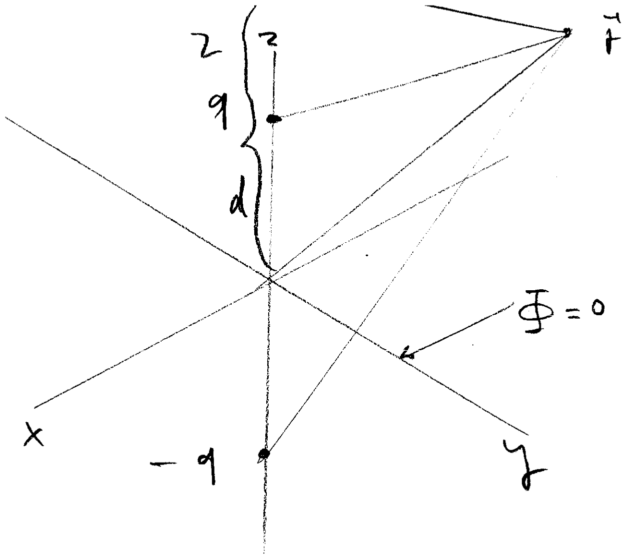
Gerçek problem

Görüntü problemi

S yüzeyi üzerinde [illegible] problemdeki potansiyeli (sınır koşulunu) sağlasınlar diye görüntü yükler koyarız.

1. Örnek: Sonsuz genişlikteki bir iletkenin karşısındaki yük





$\vec{r}\equiv(x,y,z)$ bu noktada pot.'i arıyoruz

$$\Phi(x,y,z)=\frac{+q}{\sqrt{x^2+y^2+(z-d)^2}}+\frac{-q}{\sqrt{x^2+y^2+(z+d)^2}}$$

$z \geq 0$

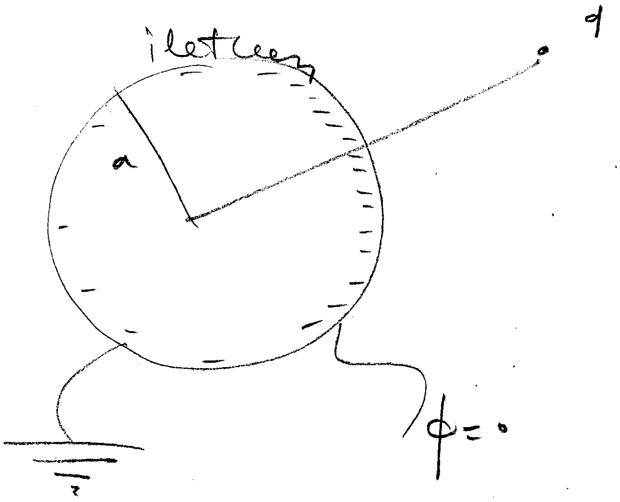
$$\Phi(x,y,0)=0$$

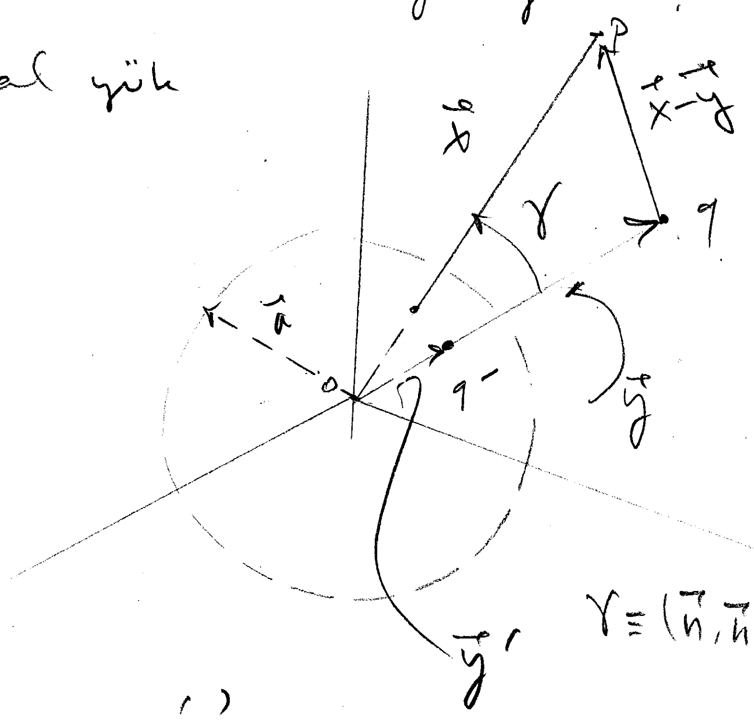
$$\left.\frac{\partial\Phi(x,y,z)}{\partial z}\right|_{z=0}=-4\pi\sigma \qquad \sigma(\vec{r})=-\frac{1}{4\pi}\left.\frac{\partial\Phi}{\partial z}\right|_0$$

$$\int_{\substack{\text{sonsuz iletken}\\ \text{yüzey}}}\sigma(\vec{r})\,da=-q$$

İletken üzerinde herhangi bir noktadaki elektrik alan yüzeye diktir.

2. Örnek : a yarıçaplı iletken küresel yüzeyin karşısında bulunan noktasal yük





$\gamma\equiv(\vec{n},\vec{n}')$

* q ve q' 'nün pot.leri küre yüzeyi üzerinde pot.'i sıfır yapacak şekilde olmalı.

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$$\Phi(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|}$$

$$1 + \frac{y^2}{a^2} - 2\frac{y}{a}\cos\gamma$$

$$1 + \frac{a^2}{y'^2} - 2\frac{a}{y'}\cos\gamma$$

$$\frac{a}{y} = \frac{y'}{a}$$

$$\vec{n} = \frac{\vec{x}}{|\vec{x}|}, \quad \vec{n}' = \frac{\vec{y}}{|\vec{y}|} = \frac{\vec{y}'}{|\vec{y}'|}$$

$$\Rightarrow \Phi(\vec{x}) = \frac{q}{x\left|\vec{n} - \frac{y}{x}\vec{n}'\right|} + \frac{q'}{y'\left|\frac{x}{y'}\vec{n} - \vec{n}'\right|}$$

$\vec{n}' - \frac{x}{y'}\vec{n}$ de yazabiliriz.

$\Phi(|\vec{x}| = a) = 0$ olmalı. $= \dfrac{q}{a\left|\vec{n} - \frac{y}{a}\vec{n}'\right|} + \dfrac{q'}{y'\left|\vec{n}' - \frac{a}{y'}\vec{n}\right|} = 0$

olması için, $\dfrac{q}{a} = -\dfrac{q'}{y'}$ ve $\dfrac{y}{a} = \dfrac{a}{y'}$ olmalıdır.

$$q' = -q\frac{y'}{a} \qquad \boxed{y' = \frac{a^2}{y}}$$

küreye göre ters noktalar

$$\Rightarrow \boxed{q' = -q\frac{a}{y}}$$

bu iki bağıntının sağlanması gerekli.

$$\Phi(\vec{x}) = \frac{q}{x\sqrt{1 + \frac{y^2}{x^2} - 2\frac{y}{x}\cos\gamma}} + \frac{q'}{y'\sqrt{1 + \frac{x^2}{y'^2} - 2\frac{x}{y'}\cos\gamma}}$$

$$= \frac{q}{x\left[1 + \frac{y^2}{x^2} - 2\frac{y}{x}\cos\gamma\right]^{1/2}} + \frac{(-qa/y)}{(a^2/y)\left[1 + x^2\frac{y^2}{a^4} - 2x\frac{y}{a^2}\cos\gamma\right]^{1/2}}$$

$$\Phi(\vec{x}) = \frac{q}{x\left[1+\frac{y^2}{x^2}-2\frac{y}{x}\cos\gamma\right]^{1/2}} - \frac{q}{a\left[1+\frac{x^2y^2}{a^4}-2\frac{xy}{a^2}\cos\gamma\right]^{1/2}}$$

$|\vec{x}| \geq a$ olduğunda geçerli.

$|\vec{x}| < a$ iç bölgede $\Phi = 0$

$\Phi(|\vec{x}|=a)=0$ olur.

yük yoğunluğu $\sigma = -\frac{1}{4\pi}\frac{\partial\Phi}{\partial x}\Big|_{x=a} = -\frac{q}{4\pi a^2}\left(\frac{a}{y}\right)\frac{\left(1-\frac{a^2}{y^2}\right)}{\left[1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\gamma\right]^{3/2}}$

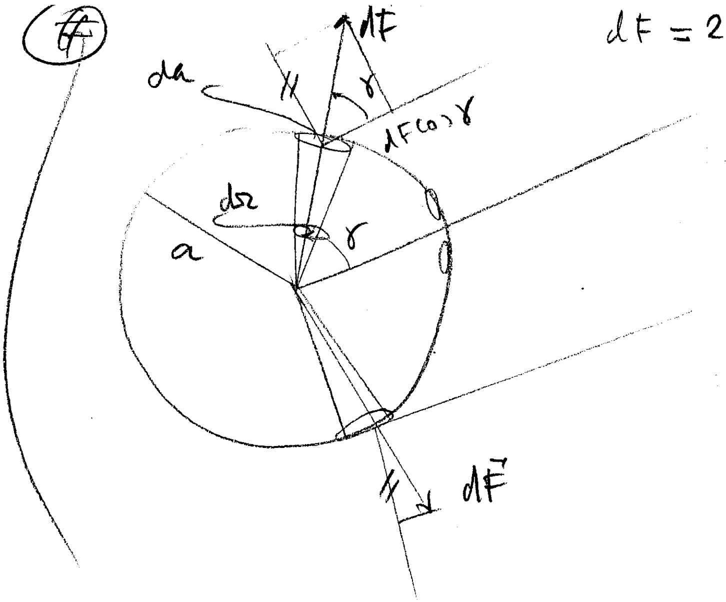
$$\int_{\text{küre yüz.}} \sigma \, da = q'$$

küre yüzeyi üzerindeki yüklerin q'ya uyguladığı kuvvet q''nün q'ya uyguladığı kuvvetle eşittir ve

$$|\vec{F}| = \frac{qq'}{|\vec{y}-\vec{y}'|^2} = \frac{qq\frac{a}{y}}{\left(y-\frac{a^2}{y}\right)^2} = \frac{q^2\frac{a}{y}}{y^2\left(1-\frac{a^2}{y^2}\right)^2}$$

$$= \frac{q^2}{a^2}\left(\frac{a}{y}\right)^3\frac{1}{\left(1-\frac{a^2}{y^2}\right)^2} \quad \text{I}$$

(f)



$dF = 2\pi\sigma^2 da$

$$\frac{da}{a^2} = d\Omega$$

küre yüzeyindeki yüklerin q'ya uyguladığı kuvvet

$$|\vec F| = \int 2\pi\sigma^2 da \cos\gamma$$

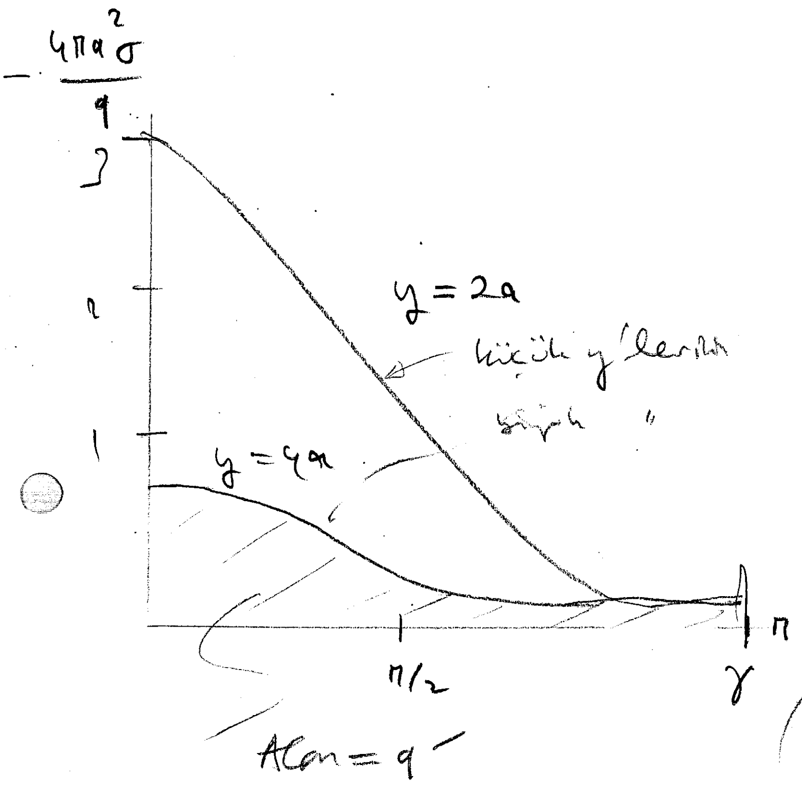
$$= 2\pi a^2 \int \sigma^2 \cos\gamma \frac{da}{a^2}$$

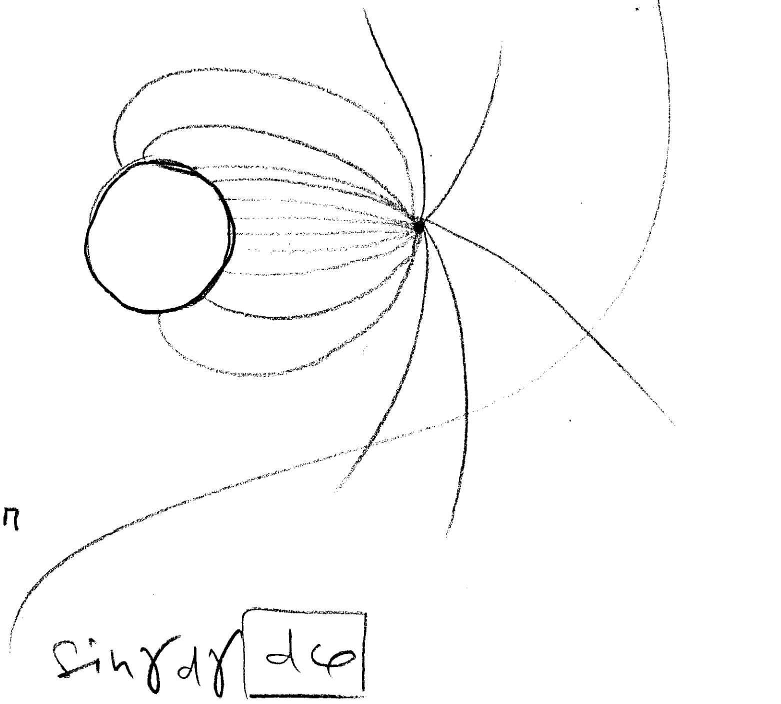
$d\Omega$

$$= 2\pi a^2 \int \sigma^2 \cos\gamma \, d\Omega$$

bu int. i almaa değerini bilmiyoruz.

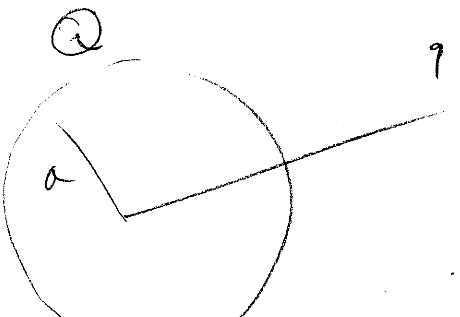
alternatif yol. ($\vec F$'yi hesaplamanın)





$$\sin\gamma \, d\gamma \, \frac{\boxed{d\varphi}}{2\pi}$$

3. Örnek Yalıtılmış iletken küre karşısındaki q yükü



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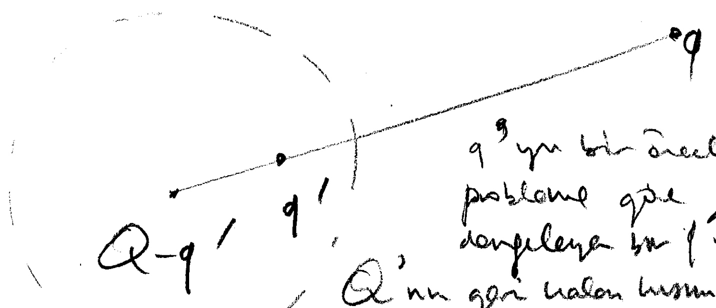
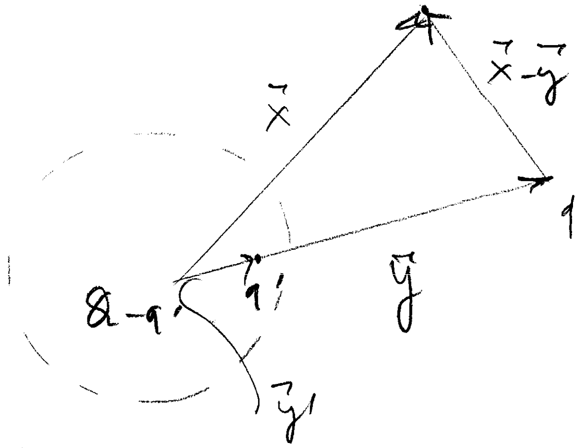


q'ya bir önceki probleme göre dengeleyen bir q' var. Q'nun geri kalan kısmını küre yüzeyi üzerinde gelişigüzel dağılmış olsun. Geri kalan yük $Q-q'$'nün merkezde toplandığını kabul edelim

$$\Phi(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|} + \frac{Q-q'}{|\vec{x}|}$$



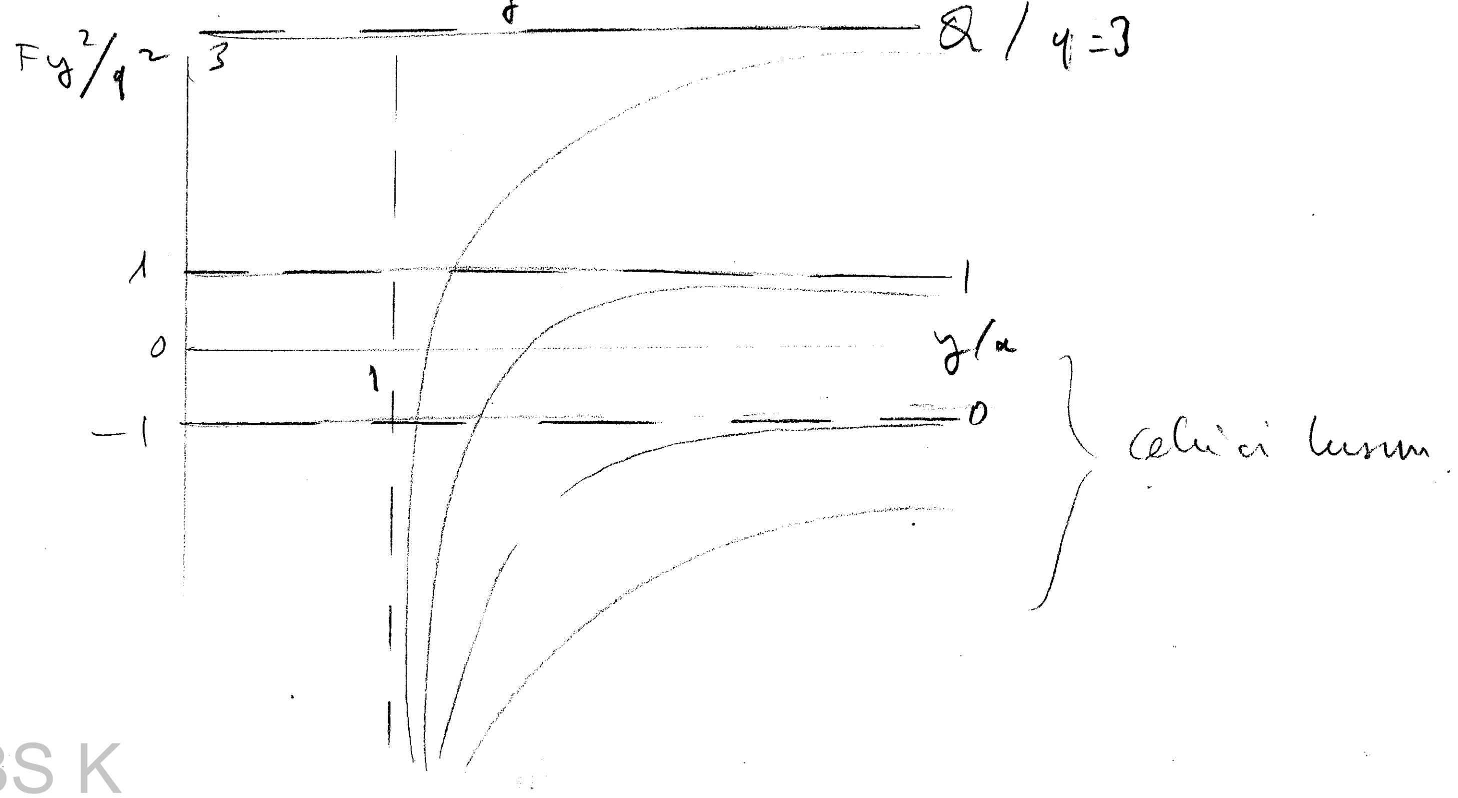
$$q' = -q\frac{a}{y} \qquad y' = \frac{a^2}{y}$$

$$\Phi(\vec{x}) = \underbrace{\frac{q}{|\vec{x}-\vec{y}|} - \frac{qa}{y\left|\vec{x}-\frac{a^2}{y^2}\vec{y}\right|}} + \frac{Q+q\frac{a}{y}}{|\vec{x}|}$$

Küre yüzeyi üzerinde bu iki terim birbirini yok eder, bu halde.

$$\vec{F} = \left\{ \frac{qq'}{|\vec{y}-\vec{y}'|^2} + \frac{q(Q-q')}{y^2} \right\} \frac{\vec{y}}{y} = \frac{q^2}{y^2} \left\{ \frac{Q}{q} - \frac{a^3(2y^2-a^2)}{y(y^2-a^2)^2} \right\} \hat{y}$$

$$y \gg a \Rightarrow |\vec{F}| \to \frac{Qq}{y^2} \Rightarrow \frac{Fy^2}{q^2} \to \frac{Q}{q}$$



2.4. Sbt pot. deki bir iletken küre karşısındaki noktasal yük.

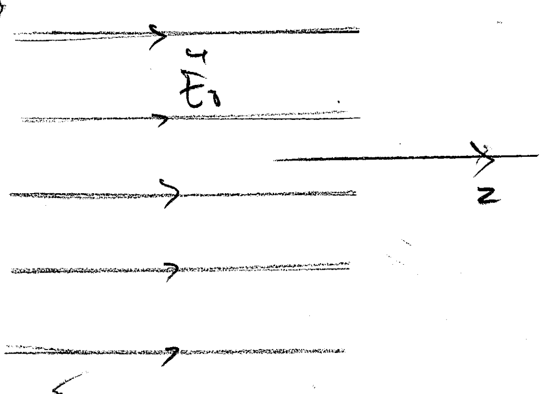
$V = \frac{Q}{a}$

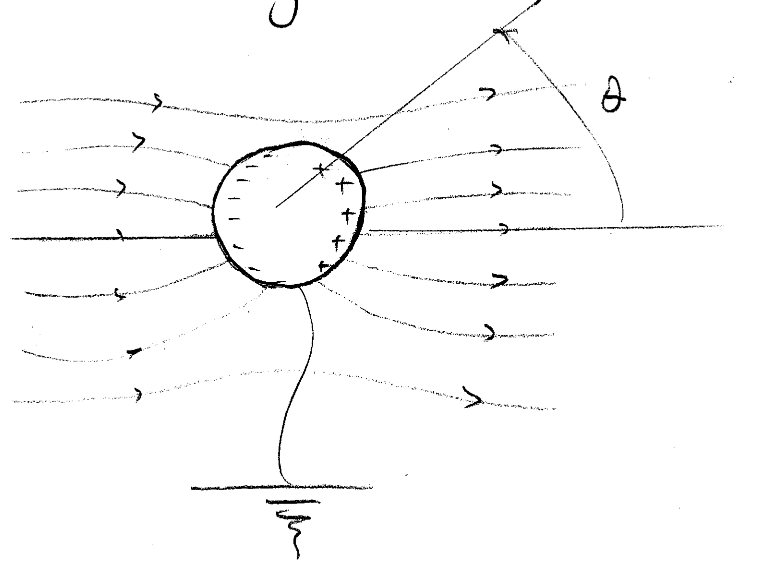
$$\Phi(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} - \frac{q\,a}{y\left|\vec{x}-\frac{a^2}{y^2}\vec{y}\right|} + \frac{V\,a}{|\vec{x}|}$$

Gerisi aynı.

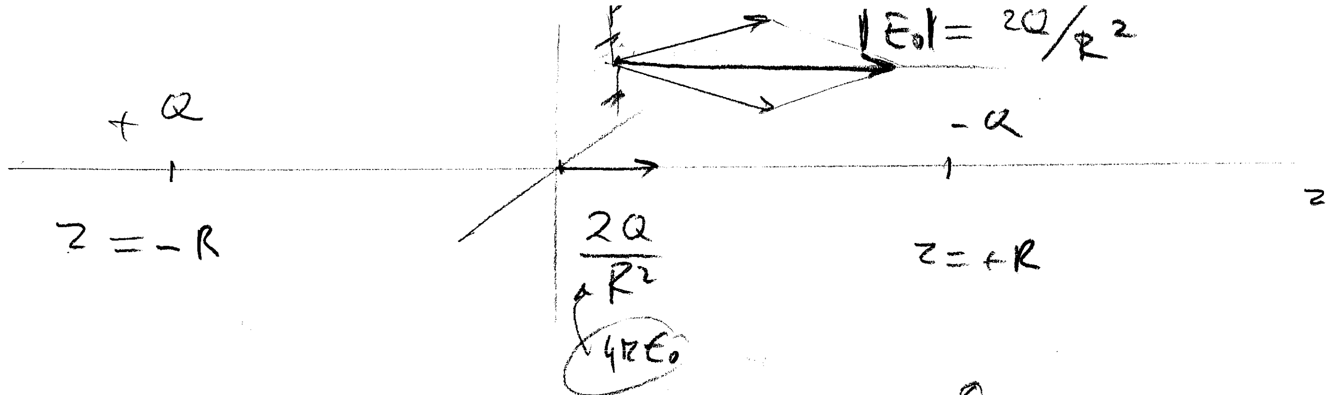
2.5. Düzgün bir elektrik alan içerisine konulmuş iletken bir küre

alalım (bu alan sonsuzda var olduğu düşünülen + ve − işaretli iki yük tarafından oluşturulmuş olduğu düşünülebilir)

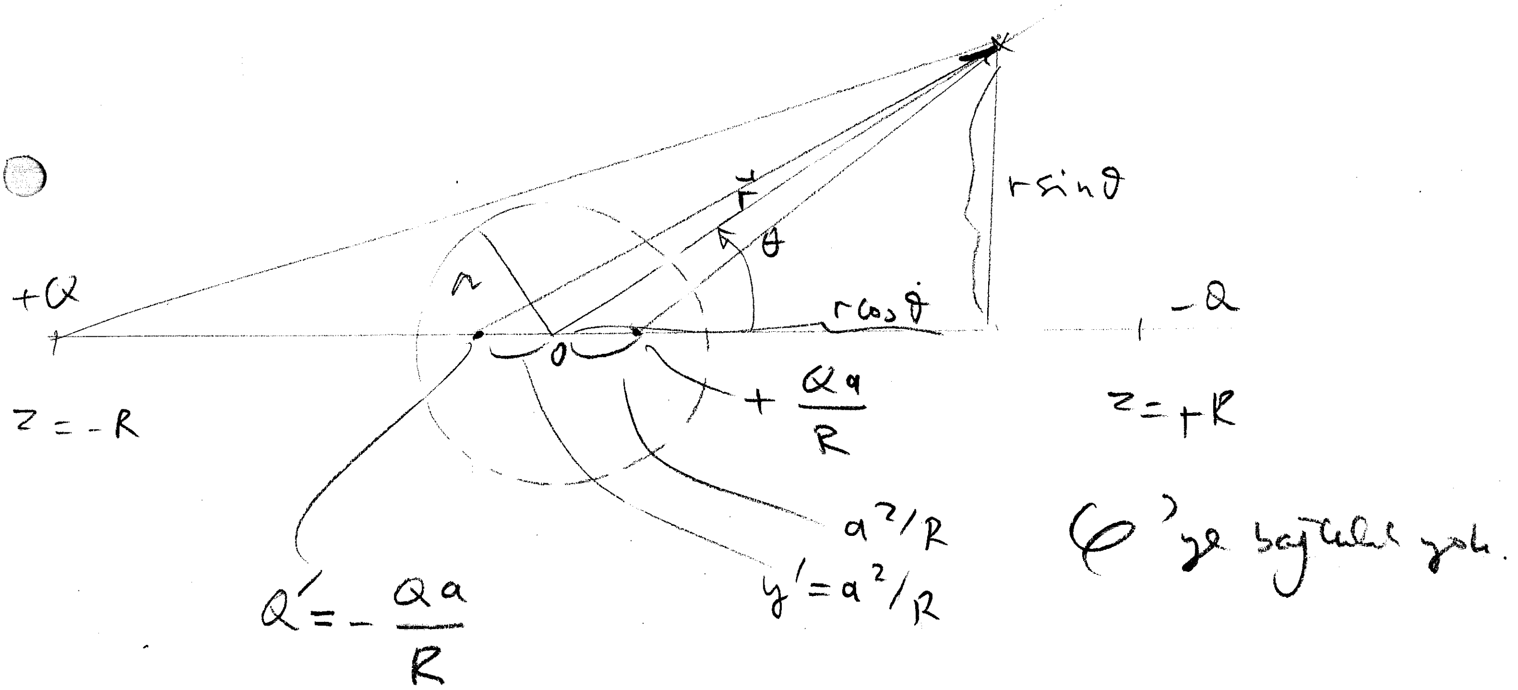




sonsuzdaki + ve − yüklerin oluşturduğu alan olarak düşünülebilir.



$R \to \infty$ iken $Q \to \infty$ gitsin öyle ki $\frac{Q}{R^2} \to$ sonlu kalsın



$z = \pm R$ deki $\mp Q$ yükleri ve bunların $z = \mp a^2/R$ deki $\mp Qa/R$ görüntülerinden kaynaklanan pot.

$$\Phi(r,\theta) = \frac{Q}{\left[(R + r\cos\theta)^2 + (r\sin\theta)^2\right]^{1/2}} - \frac{Q}{\left[(R - r\cos\theta) + (r\sin\theta)^2\right]^{1/2}}$$

$$- \frac{aQ}{R\left[\left(\frac{a^2}{R} + r\cos\theta\right)^2 + (r\sin\theta)^2\right]^{1/2}} + \frac{aQ}{R\left[\left(\frac{a^2}{R} - r\cos\theta\right)^2 + (r\sin\theta)^2\right]^{1/2}}$$

$$= \frac{Q}{\left[R^2 + r^2 + 2rR\cos\theta\right]^{1/2}} - \frac{Q}{\left[R^2 + r^2 - 2rR\cos\theta\right]^{1/2}} - \frac{Q}{\left[\frac{R^2 r^2}{a^2} + a^2 + 2rR\cos\theta\right]^{1/2}}$$

$$+ \frac{Q}{\left[\frac{R^2 r^2}{a^2} + a^2 - 2rR\cos\theta\right]^{1/2}}$$

$$\Phi(r,\vartheta) = \frac{Q}{R}\left[1+\left(\frac{r}{R}\right)^2+2\frac{r}{R}\cos\vartheta\right]^{-1/2} - \frac{Q}{R}\left[1+\left(\frac{r}{R}\right)^2-\frac{2r}{R}\cos\vartheta\right]^{-1/2}$$

$$-\frac{Q}{R}\left[\left(\frac{r}{a}\right)^2+\left(\frac{a}{R}\right)^2+2\frac{r}{R}\cos\vartheta\right]^{-1/2} + \frac{Q}{R}\left[\left(\frac{r}{a}\right)^2+\left(\frac{a}{R}\right)^2-2\frac{r}{R}\cos\vartheta\right]^{-1/2}$$

$\left(r/R\right), \left(\frac{a}{R}\right) \ll 1$ ihmal edip seriye aç.

$$= \frac{Q}{R}\left(\cancel{1} - \frac{r}{R}\cos\vartheta + \dots\right) - \frac{Q}{R}\left(\cancel{1} + \frac{r}{R}\cos\vartheta + \dots\right)$$

$$-\frac{Q}{R}\frac{1}{(r/a)}\left(\cancel{1} - \frac{a^2}{rR}\cos\vartheta + \dots\right) + \frac{Q}{R}\frac{1}{(r/a)}\left(\cancel{1} + \frac{a^2}{rR}\cos\vartheta + \dots\right)$$

$$= -\underbrace{\frac{2Q}{R^2}}_{E_0} r\cos\vartheta + \frac{2Q}{R^2}\frac{a^3}{r^2}\cos\vartheta$$

üniform alan

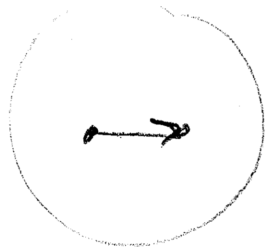
$$= -E_0\left(1 - \frac{a^3}{r^3}\right) r\cos\vartheta$$

yüzeyde indüklenmiş yük yoğunluğundan dolayı

merkezde oluşan dipol mom.

$$= -E_0 z + \frac{\overbrace{E_0 a^3}^{|\vec{D}|}}{r^3} z$$

E_0'dan dolayı pot.



$$D = Q\frac{a}{R}\cdot\frac{2a^2}{R} = \overbrace{\frac{2Q}{R^2}}^{E_0} a^3$$

$$\sigma = -\frac{1}{4\pi}\left.\frac{\partial\Phi}{\partial r}\right|_{r=a} = -\frac{1}{4\pi}\left(-E_0\cos\vartheta - \frac{2a^3}{r^3}E_0\cos\vartheta\right)\Big|_{r=a} = \frac{3E_0}{4\pi}\cos\vartheta$$

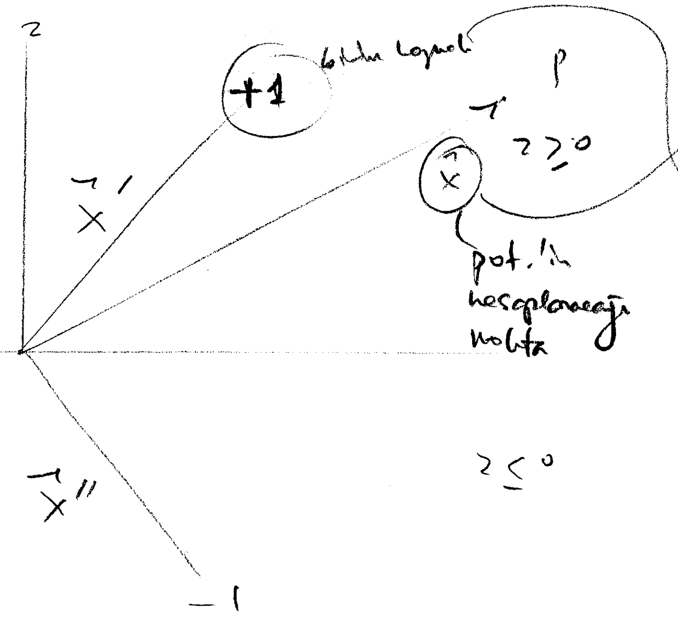
yüzey integrali "sıfır". (:) topraklanmış ve yalıtılmış küreler aralarında fark yok.

2.6. Küre için Green Fonk.'u : Pot. için

Genel çözüm. {Birim kaynak ve homojen sınır şartlarını sağlamak için seçilmiş bunun görüntüsü (ya da görüntüleri) Dirichlet ya da Neumann sınır şartları için uygun Green fonk.'undan başka bir şey değildi. (nden kaynaklanan pot.)

Not: (Hatırlatma)

Düzlem için Green fon.



$\vec{x} \equiv (x, y, z)$

$\vec{x}' \equiv (x', y', +z')$

$\vec{x}'' \equiv (x', y', -z')$

$$G_D(\vec{x}, \vec{x}') = \frac{+1}{|\vec{x} - \vec{x}'|} + F(\vec{x}, \vec{x}')$$

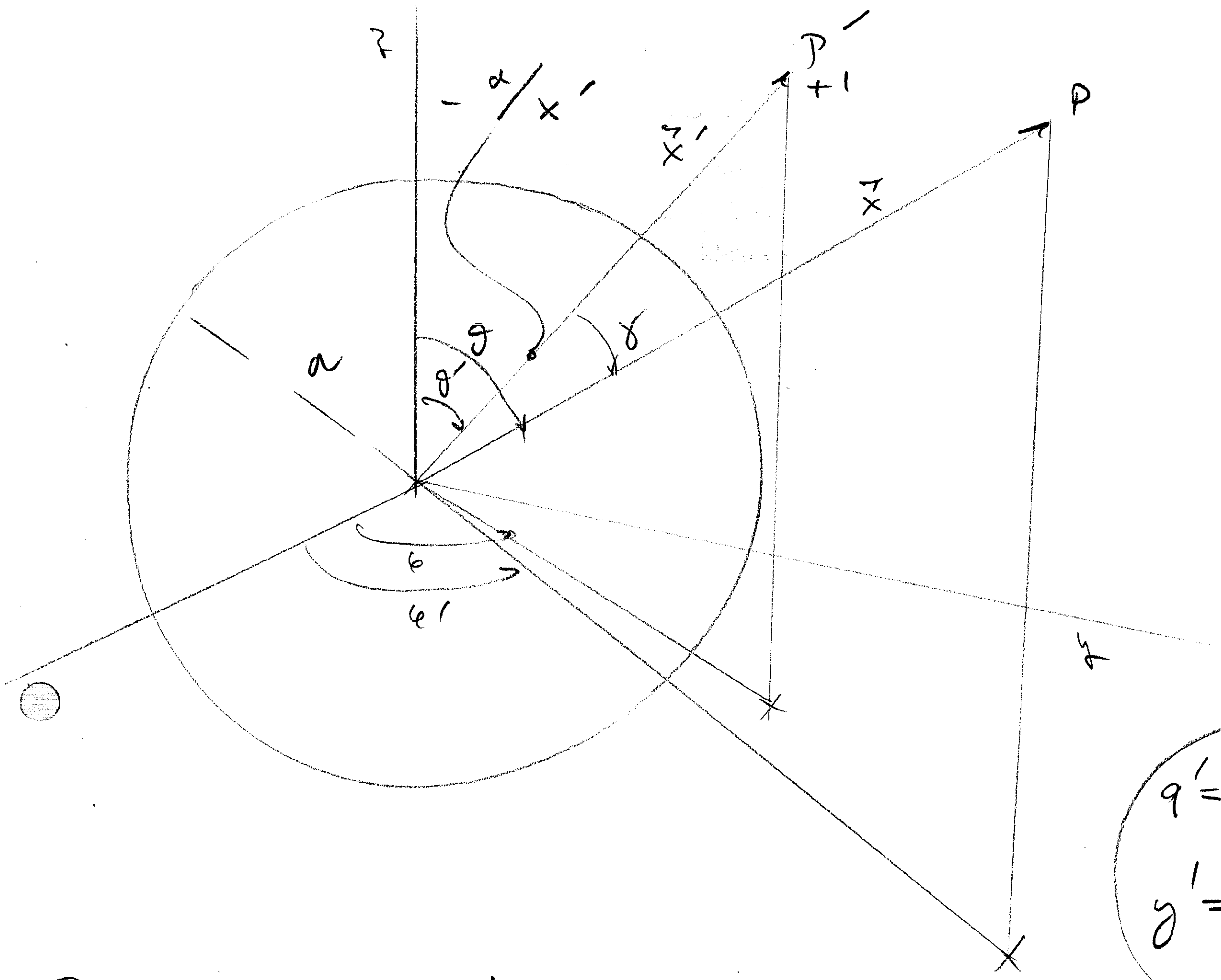
düzlem için

$$F(\vec{x}, \vec{x}') = -\frac{1}{|\vec{x} - \vec{x}''|}$$

$$= \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{-1/2} - \left[(x-x')^2 + (y-y')^2 + (z+z')^2\right]^{-1/2}$$

$$G_D(x, y, z=0, \vec{x}') = 0 \quad \text{(Düzlem üzerinde)}$$

Küre için Green fonk.'u :



$$q' = -\frac{a}{y}\,q$$

$$y' = \frac{a^2}{y}$$

ini.

$$G(\vec{x},\vec{x}') = \frac{1}{|\vec{x}-\vec{x}'|} - \frac{a}{x'\,\left|\vec{x} - \frac{a^2}{x'^2}\vec{x}'\right|}$$

$$= \left[x^2 + x'^2 - 2xx'\cos\gamma\right]^{-1/2} - \frac{a}{x'}\left[x^2 + \frac{a^4}{x'^2} - 2\frac{a^2}{x'^2}x\cos\gamma\right]^{-1/2}$$

$$= \left[\qquad\qquad\right]^{-1/2} - \left[\left(\frac{xx'}{a}\right)^2 + a^2 - 2xx'\cos\gamma\right]^{-1/2}$$

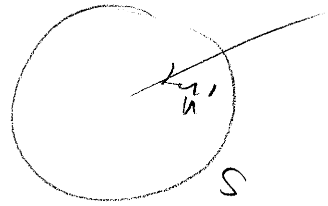
$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi')$$

$$\Phi(\vec{x}) = \int_V \rho(\vec{x}')\,G_D(\vec{x},\vec{x}')\,d^3x'$$

$$+ \frac{1}{4\pi}\oint_S \Phi(\vec{x}')\frac{\partial G_D}{\partial n'}\,da'$$

Dirichlet

$\vec{n}'$ 'nün yönü hacimden yukarı doğru. Küre yüzeyinin dışı hacim olarak alındığına göre $\vec{n}'$ 'nün yönü merkeze doğru

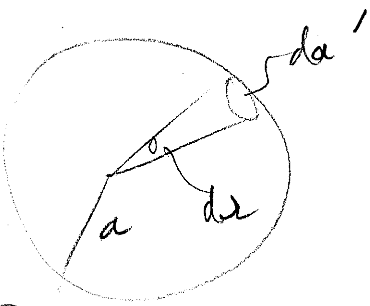


$$\frac{\partial G_D}{\partial n'} = -\left.\frac{\partial G_D}{\partial x'}\right|_{x'=a}$$

$$\frac{\partial G_D}{\partial x'} = -\frac{1}{2}\,\frac{2x' - 2x\cos\gamma}{(\quad)^{3/2}} + \frac{1}{2}\,\frac{\frac{2x^2x'}{a^2} - 2x\cos\gamma}{((\quad))^{3/2}}$$

$$\left.\frac{\partial G_D}{\partial x'}\right|_{x'=a} = \frac{x^2-a^2}{a\left(x^2+a^2-2xa\cos\gamma\right)^{3/2}}$$

[2.5. yüzeysel yük yoğunluğu] olduğuna dikkat et!



$$d\Omega = \frac{da'}{a^2}$$

dışarıda çok yük ile

$$\frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} = 0 \text{ dır.}$$

$$\nabla^2\Phi = 0$$

$$\Phi(x,\theta,\varphi) = \frac{1}{4\pi}\oint \Phi(a,\theta',\varphi') \times \frac{x^2-a^2}{a\left(x^2+a^2-2xa\cos\gamma\right)^{3/2}}\, a^2\, d\Omega$$

$$= \frac{a(x^2-a^2)}{4\pi}\int \Phi(a,\theta',\varphi')\,\frac{1}{(\quad)}$$

$\sin\theta'\, d\theta'\, d\varphi'$

Kürenin iç bölgesindeki pot. problemi ise yüzeyle dik türev dışa doğru

$$-\frac{\partial G_D}{\partial x} \longrightarrow \frac{\partial G_D}{\partial x} \text{ yazarız.}$$

yine $\dfrac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|}$ sıfır olabilir.

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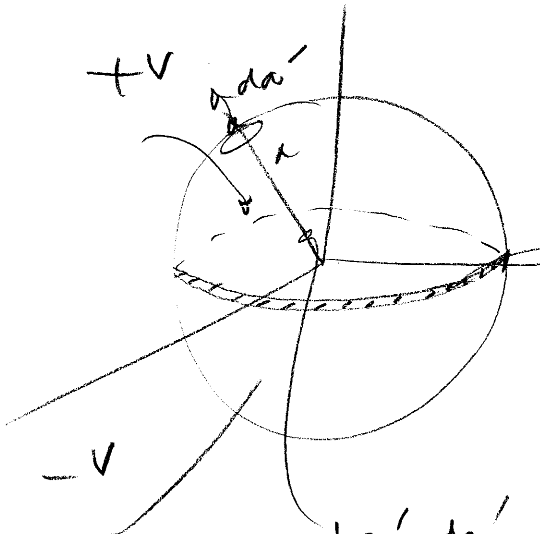
2.7. Farklı potansiyellerdeki yarı-küreli iletkenler



$$\Phi(a,\theta,\varphi) = \begin{cases} +V & 0 \le \theta \le \pi/2 \\ -V & \frac{\pi}{2} \le \theta < \pi \end{cases}$$

Dış bölgedeki $\Phi = ?$

$$d\Omega' = \frac{da'}{a^2} = \sin\theta' d\theta' d\varphi' = -d\varphi' d(\cos\theta')$$

$$\Phi(x,\theta,\varphi) = \frac{1}{4\pi} \underset{\text{küreyüz.}}{\oint} \underbrace{\Phi(a,\theta',\varphi')}_{V} \frac{\partial G_D}{\partial n'} da' \qquad a^2 da' = d\Omega$$

$$\frac{(x^2 - a^2)}{\cancel{x}(\quad)^{3/2}} \, a^{\cancel{2}}$$

$$(\theta' \to \pi - \theta', \varphi \to \varphi' + \pi)$$

$$= \frac{V}{4\pi} \int_0^{2\pi} d\varphi' \Big\{ \underbrace{\int_1^0 - d(\cos\theta')}_{+V} - \underbrace{\int_0^{-1} - d(\cos\theta')}_{-V} \Big\} \frac{a(x^2 - a^2)}{(\quad)^{3/2}}$$

$$= \frac{V}{4\pi} \int_1^{2\pi} d\varphi' \left[\int_0^1 d(\cos\theta') \frac{a(x^2 - a^2)}{(\;-\;)^{3/2}} - \int_0^1 d(\cos\theta') \frac{a(x^2 - a^2)}{(\;+\;)^{3/2}} \right]$$

$\cos\gamma = \cos\theta'$ $(\theta = 0)$ özel durumu hesaplayalım!

$$= \frac{Va}{4\pi} (z^2 - a^2) \int_0^{2\pi} d\varphi' \int_0^1 d(\cos\theta') \Big[(z^2 + a^2 - 2az\cos\theta')^{-3/2} + (z^2 + a^2 + 2az\cos\theta')^{-3/2} \Big]$$

$$u = a^2 + z^2 - 2azt \qquad du = -2az\, dt$$

$$\int_0^1 dt \frac{1}{[a^2+z^2-2azt]^{3/2}} = -\frac{1}{2az}\int du\, u^{-3/2} = \frac{1}{az}\frac{1}{(z^2+a^2-2azt)^{1/2}}\Bigg|_0^1$$

$$= \frac{1}{az}\left[\frac{1}{(z-a)} - \frac{1}{\sqrt{z^2+a^2}}\right]$$

Öbür int. $= \frac{-1}{az}\left[\frac{1}{z+a} - \frac{1}{\sqrt{z^2+a^2}}\right]$

$$\Rightarrow \Phi = \frac{V}{2}\frac{a(z^2-a^2)}{az}\left[\frac{1}{z-a} - \frac{1}{\sqrt{z^2+a^2}} + \frac{1}{z+a} - \frac{1}{\sqrt{z^2+a^2}}\right]$$

$$= \frac{V}{\cancel{2}}\frac{a(z^2-a^2)}{az}\left[\frac{\cancel{2}z}{z^2-a^2} - \frac{\cancel{2}}{\sqrt{z^2+a^2}}\right]$$

$$= V\left[1 - \frac{z^2-a^2}{z\sqrt{z^2-a^2}}\right]$$

$$\Phi(z=a) = V$$

$$\frac{1}{(x^2+a^2)^{3/2}\left(1 \mp \frac{2ax}{x^2+a^2}\cos\gamma\right)^{3/2}}$$ terimi

b

$x \gg a \Rightarrow b = \frac{ax}{x^2+a^2} < 1/2$ $\qquad (1+a)^m = 1 + ma + \frac{m(m-1)}{2!}a^2 + \dots$

kulla!

$$\left[(1-2\alpha\cos\gamma)^{-3/2} - (1+2\alpha\cos\gamma)^{-3/2} \right]$$

$$= \left[\cancel{1} + \frac{3}{2}\, 2\alpha\cos\gamma + \frac{3}{2}\,\frac{5}{2}\,\frac{1}{2}\, 4\alpha^2\cos^2\gamma + \frac{1}{3.2}\,\frac{3}{2}\,\frac{5}{2}\,\frac{7}{2}\, 8\alpha^3\cos^3\gamma + \dots \right]$$

$$- \left[\cancel{1} - \frac{3}{2}\, 2\alpha\cos\gamma + \frac{3}{2}\,\frac{5}{2}\,\frac{1}{2}\, 4\alpha^2\cos^2\gamma + \dots \right]$$

$$= 6\alpha\cos\gamma + 35\alpha^3\cos^3\gamma$$

- $$\int_0^{2\pi} d\varphi' \int_0^1 dt \left[t\cos\theta + \sin\theta\,(1-t^2)^{1/2}\cos(\varphi-\varphi') \right] = \dots$$

2.8
Dik Fonksiyonlara açılım cinsinden potansiyel çözümü

ξ : (a,b) aralığında olsun

"seçilecek olan fonk. kümesi problemin simetrisine bağlıdır."

- $\{ U_n(\xi) \mid n=1,2,\dots \}$

(a,b) aralığında diktirler demek,

$$\int_a^b U_n^*(\xi)\, U_m(\xi)\, d\xi = \begin{cases} 0 & m \neq n \\ \text{Sonlu} & m = n \end{cases}$$

bu durumda $\{U_n(\xi)\}$ kümesine ortonormal fonk.lar kümesi denir.

$$\int_a^b U_n^*(\xi)\, U_m(\xi)\, d\xi = \delta_{nm} \quad \text{ortonormal.}$$

← Diklik koşulu.

* (a,b) aralığında karesi integrallenebilir her $f(\xi)$ fonk.'u

$\left[\int_a^b d\xi \, |f(\xi)|^2 < \infty \right]$,

∞ olduğunda en iyi

$$f(\xi) \longleftrightarrow (=) \sum_{n=1}^{M} a_n u_n(\xi)$$

ortonormal fonk.lar cinsinden seriye açılabilir

$$a_n = \int_a^b u_n^*(\xi) f(\xi) d\xi$$

olduğu kolayca gösterilebilir. Bu durumda,

$$f(\xi) = \sum_n \left\{ \int_a^b u_n^*(\xi') f(\xi') d\xi' \right\} u_n(\xi)$$

$$= \int_a^b d\xi' \Big[\underbrace{\sum_n u_n^*(\xi') u_n(\xi)}_{\delta(\xi'-\xi)} \Big] f(\xi')$$

tamlık yada kapanış bağıntısı.

Mat. Fiz. de bilinen tüm ort. fonk. lar tam'dır.

En iyi bilinen ortogonal fonk.'lar sinüs ve kosinüsler

$$\sqrt{\frac{2}{a}} \sin \frac{2\pi m x}{a} \quad , \quad \sqrt{\frac{2}{a}} \cos \frac{2m\pi x}{a}$$

m negatif olmayan bir tamsayı

$1/\sqrt{a}$ $m=0$ için

2.33 e eşdeğer seri, $(-a/2, +a/2)$ aralığında

$$f(x) = \underbrace{a_0 \frac{1}{\sqrt{a}}}_{\frac{A_0}{2}} + \sum_{n=1}^{\infty} \overbrace{a_n \sqrt{\frac{2}{a}}}^{A_n} \cos \frac{2n\pi x}{a} + \sum_{n=1}^{\infty} \overbrace{b_n \sqrt{\frac{2}{a}}}^{B_n} \sin \frac{2\pi n x}{a}$$

1. $$\int_{-a/2}^{+a/2} dx \frac{1}{\sqrt{a}} f(x) = a_0 \left(\frac{1}{\sqrt{a}}\right)^2 \int_{-a/2}^{+a/2} dx = a_0$$

$$\int_{-a/2}^{+a/2} \sqrt{\frac{2}{a}} f(x) \cos \frac{2m\pi x}{a} dx = a_m \underbrace{\int_{-a/2}^{+a/2} dx \left(\sqrt{\frac{2}{a}} \cos \frac{2n\pi x}{a}\right)^2}_{1}$$

$$a_m = \sqrt{\frac{2}{a}} \int f(x) \cos \frac{2m\pi x}{a} dx$$

$$b_m = \sqrt{\frac{2}{a}} \int f(x) \sin \frac{2\pi m x}{a} dx$$

$$\begin{cases} A_m = \dfrac{2}{a} \displaystyle\int \dots \\ B_m = \dfrac{2}{a} \displaystyle\int \dots \\ A_0 = \dfrac{2}{a} \displaystyle\int f(x)\, dx \end{cases} \qquad 2.37.$$

Aralık bir boyutta daha fazla ortonormal set tarafından gerilirse,

$$f(\xi, \zeta) = \sum_n \sum_m a_{nm} U_n(\xi) V_m(\zeta)$$

$$a_{nm} = \int_a^b \int_c^d d\xi\, d\zeta\, U_n^*(\xi) V_m^*(\zeta) f(\xi, \zeta)$$

(a,b) aralığı $\to \infty$ olursa $U_n(\xi)$ fonk. ları sürekli fonk. lar olurlar.

$$\begin{cases} \delta_{nm} \to \delta(\quad) \\ \sum \to \int \end{cases}$$

Örn Fourier integrali

$$U_m(x) = \frac{1}{\sqrt{a}} e^{i(2\pi m x/a)}$$

komp. exp. lerin kümesini ele alalım.

$$f(x) = \frac{1}{\sqrt{a}} \sum_{m=-\infty}^{+\infty} A_m e^{i(2\pi m x/a)}$$ içine koyabiliriz.

açılımı ile $(-a/2, a/2)$ aralığında $m = 0, \pm 1, \pm 2, \ldots$

$$A_m = \frac{1}{\sqrt{a}} \int_{-a/2}^{+a/2} e^{-i(2\pi m x'/a)} f(x')\,dx'$$

$a \to \infty$ olduğunda
$$\begin{cases} \frac{2\pi m}{a} \to k \quad \text{sürekli} \\ \sum_m \to \int_{-\infty}^{+\infty} dm = \frac{a}{2\pi} \int_{-\infty}^{+\infty} dk \\ A_m \to \sqrt{\frac{2\pi}{a}}\, A(k) \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(k)\, e^{ikx}\, dk \; ; \quad A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-ikx} f(x)\, dx$$

ortogonallik, $\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(k-k')x}\,dx = \delta(k-k')$

tamlık $\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ik(x-x')}\,dk = \delta(x-x')$

2.9. Laplace denk.'nin kartezyen koordinatlarda değişkenlerine ayrılarak çözümü

$\vec{\nabla}^2 \Phi(x,y,z) = 0 \qquad \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$

$$\Phi(x,y,z) = X(x)Y(y)Z(z)$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{f(x)} = \underbrace{-\frac{1}{Y}\frac{d^2Y}{dy^2} - \frac{1}{Z}\frac{d^2Z}{dz^2}}_{f(y,z)}$$

aynı sabit'e eşit olmaları gerekir.

$f(x) \downarrow$ sbt $-\alpha^2$

$$\boxed{\frac{1}{X}\frac{d^2X}{dx^2} = -\alpha^2}$$

$\alpha^2 > 0$

$$\underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{-\beta^2} = -\frac{1}{Z}\frac{d^2Z}{dz^2} + \alpha^2$$

BS K

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = \alpha^2+\beta^2 = \gamma^2$$

$$X(x) = Ae^{i\alpha x} + Be^{-i\alpha x}$$

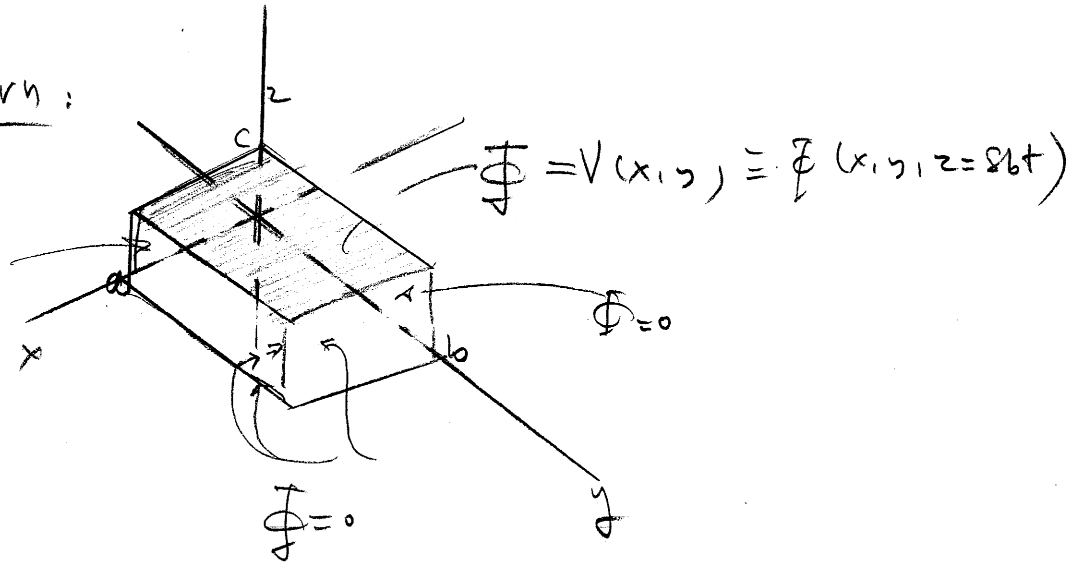
$$Y(y) = Ce^{i\beta y} + De^{-i\beta y}$$

$$Z(z) = Ee^{\gamma z} + Fe^{-\gamma z}$$

$$\Phi(x,y,z) = (\quad)\times(\quad)\times(\quad)$$

$$\overset{\text{d}}{=} e^{\pm i\alpha x}\; e^{\pm i\beta y}\; e^{\pm\sqrt{\alpha^2+\beta^2}\,z}$$

Örn:



$$\Phi(0,y,z) = \Phi(a,y,z) = 0$$

$$X(0) = A+B = 0 \Rightarrow A=-B$$

$$X(x) = A\left(e^{i\alpha x} - e^{-i\alpha x}\right) = 2iA\sin\alpha x$$

$$X(a) = 0 \Rightarrow \alpha a = n\pi \quad n=1,2,3,\dots$$

$$Y(0)=0 \Rightarrow D=-C$$

$$Y(y)=D\left(e^{i\beta y}-e^{-i\beta y}\right)=2iD\sin\beta y$$

$$Y(b)=0 \Rightarrow \beta b = m\pi \qquad m=1,2,3,\ldots$$

$$Z(0)=0 \Rightarrow E=-F \qquad Z(z)=2E\sinh\gamma z$$

$$\Phi_{nm}(x,y,z)=A\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\sinh\gamma_{nm}z$$

$$\gamma_{nm}=\sqrt{\frac{n^2\pi^2}{a^2}+\frac{m^2\pi^2}{b^2}}$$

$$\Phi(x,y,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{nm}$$ cinsinden seriye açılabilir.

$$\Phi(x,y,z)=\sum_n\sum_m A_{nm}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\sinh\gamma_{nm}z$$

$$\Phi=V(x,y) \qquad z=c \text{ de}$$

$$C_{nm}$$

$$V(x,y)=\sum_n\sum_m A_{nm}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}\sinh\gamma_{nm}c$$

$$\int_0^a dx\int_0^b dy\, V(x,y)\sin\frac{n'\pi x}{a}\sin\frac{m'\pi y}{b}=\sum_n\sum_m C_{nm}$$

$$\times\underbrace{\int_0^a dx\sin\frac{n\pi x}{a}\sin\frac{n'\pi x}{a}}_{(a/2)\delta_{nn'}}\underbrace{\int_0^b dy\sin\frac{m\pi y}{b}\sin\frac{m'\pi y}{b}}_{(a/2)\delta_{mm'}}$$

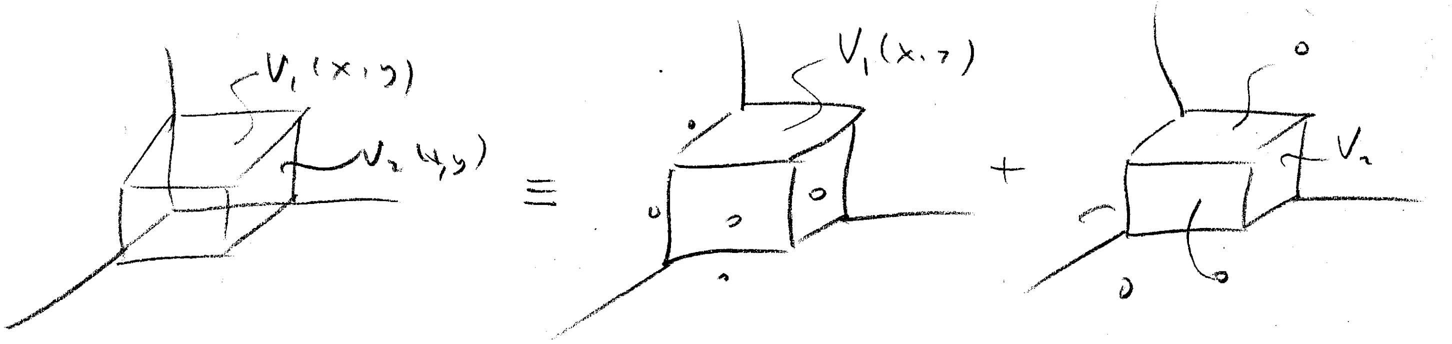
$$\int_0^a \sin^2\frac{n\pi x}{a}\,dx = \frac{a}{n\pi}\int_0^{n\pi}\frac{1}{2}\left(1-\cos 2w\right)dw$$

$$= \frac{a}{\cancel{n\pi}}\,\frac{\cancel{n\pi}}{2} = \frac{a}{2}$$

$$\int_0^a\int_0^b dx\,dy\,V(x,y)\sin\frac{n'\pi x}{a}\sin\frac{m'\pi y}{b} = \frac{ab}{4}\,C_{n'm'}$$

$A_{n'm'}\sinh\gamma_{n'm'}c$

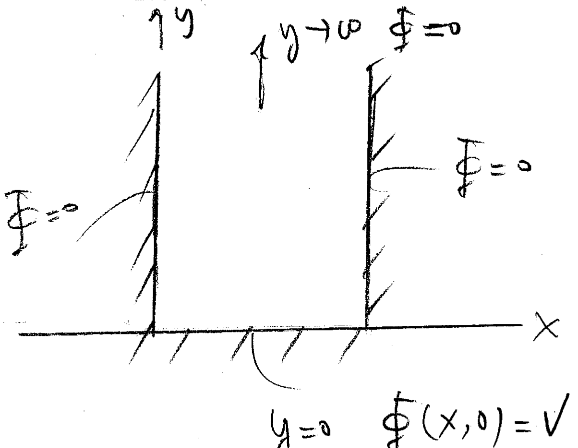
$$A_{nm} = \frac{4}{ab\sinh\gamma_{nm}c}\int_0^a\int_0^b dx\,dy\,V(x,y)\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$



lineer kombinasyonlar, süperpozisyon ilkesi geçerli.

2.10. İki boyutta Laplace denk.'i.

$0 \le x \le a$, $y \ge 0$ bölgesinde pot. aranmaktadır.



$$\frac{\partial^2\Phi(x,y)}{\partial x^2} + \frac{\partial^2\Phi(x,y)}{\partial y^2} = 0$$

$$\Phi(x,y) = X(x)\,Y(y)$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{-\alpha^2} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{+\alpha^2} = 0$$ olmalı.

bu uygun sınır koşulları için. x de sinüsoidal olması için.

$$X(x) = A e^{i\alpha x} + B e^{-i\alpha x}$$

$$Y(y) = C e^{\alpha y} + D e^{-\alpha y}$$

$$X(0) = A + B = 0 \qquad B = -A$$

$$X(x) = A' \sin \alpha x$$

$$X(a) = 0 \Rightarrow \alpha a = n\pi \qquad \alpha_n = \frac{n\pi}{a}$$

$$Y(y \to \infty) = 0 \Rightarrow C = 0$$

$$Y(y) = D e^{-\alpha y}$$

$$\Phi(x,y) = \sum_n a_n \sin \frac{n\pi x}{a} e^{-n\pi y/a}$$

4. sınır koşulu ile bulabiliriz bunu da,

$$\Phi(x,0) = V(x) = \sum_n a_n \sin \frac{n\pi x}{a} = V$$

$$a_n = \frac{2}{a} \int_0^a V(x) \sin \frac{n\pi x}{a}\, dx$$

$$V(x) = V = \text{slt} = \frac{2V}{a} \int_0^a \sin \frac{n\pi x}{a}\, dx$$

BS K

$$= \frac{2V}{\cancel{a}} \frac{\cancel{a}}{n\pi} \int_0^{n\pi} \sin u \, du = \frac{2V}{n\pi} \left(-\cos u \right) \Big|_0^{n\pi}$$

$$= \frac{2V}{n\pi} \left[1 - (-1)^n \right] = \begin{cases} 4V/n\pi & n \text{ tek} \\ 0 & n \text{ çift.} \end{cases}$$

$$\Phi(x,y) = \frac{4V}{\pi} \underbrace{\sum_{n\,\text{tek}} \frac{1}{n} \sin \frac{n\pi x}{a} e^{-n\pi y/a}}$$

$$e^{-\pi y/a} \sin \frac{\pi x}{a} + e^{-3\pi y/a} \sin \frac{3\pi x}{a} + \cdots$$

(i)

$\frac{\pi y}{a} \geq 1 \Rightarrow y \geq \frac{a}{\pi}$ için Φ çözümü

$$\Phi \simeq \frac{4V}{\pi} e^{-\pi y/a} \sin \frac{\pi x}{a}$$

(ii)

$$\operatorname{Im} e^{i\theta} = \sin\theta$$

$$\Phi(x,y) = \frac{4V}{\pi} \operatorname{Im} \sum_{n\,\text{tek}} \frac{1}{n} e^{-\frac{n\pi y}{a}} e^{\pm i \frac{n\pi x}{a}}$$

$$= \frac{4V}{\pi} \operatorname{Im} \left\{ \sum \frac{1}{n} e^{in\pi/a\,(x+iy)} \right\}$$

$$Z = e^{\frac{i\pi}{a}(x+iy)} \Rightarrow \Phi = \frac{4V}{\pi} \operatorname{Im} \left\{ \sum \frac{1}{n} Z^n \right\}$$

BS K

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \cdots \quad \text{Taylor serisi}$$

$$\ln(1-z) = -z - \frac{z^2}{2} - \frac{z^3}{3} - \frac{z^4}{4} - \cdots$$

$$\left(\ln(1+z)\right)' = \frac{1}{1+z}\Big|_{z=0} = 1$$

$$\left(\ln(1+z)\right)'' = \frac{-1}{(1+z)^2}\Big|_{z=0} = -1$$

$$\ln(1+z) - \ln(1-z) = \ln\left(\frac{1+z}{1-z}\right)$$

$$= 2z + 2\frac{z^3}{3} + \cdots$$

$$= 2\sum_{n\ \text{tek}} \frac{z^n}{n}$$

$$\Phi = \frac{4V}{\pi} \operatorname{Im}\left\{\frac{1}{2}\ln\frac{1+z}{1-z}\right\} = \frac{2V}{\pi} \operatorname{Im} \ln\frac{1+z}{1-z}$$

$$w = x + iy = \rho e^{i\theta}; \quad \rho = \sqrt{x^2+y^2}; \quad \operatorname{tg}\theta = \frac{y}{x}$$

$$\ln w = \ln\rho + i\theta$$

$$\operatorname{Im} \ln w = \theta = \operatorname{tg}^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{\operatorname{Im} w}{\operatorname{Re} w}\right)$$

$$\frac{1+z}{1-z} = \frac{(1+z)(1-z^*)}{(1-z)(1-z^*)} = \frac{1-|z|^2+2i\operatorname{Im} z}{1+|z|^2-2\operatorname{Re} z}$$

$$\tan^{-1}\left\{\frac{2\operatorname{Im} z/(1+|z|^2-2\operatorname{Re} z)}{1-|z|^2/(1+|z|^2-2\operatorname{Re} z)}\right\} = \tan^{-1}\left(\frac{2\operatorname{Im} z}{1-|z|^2}\right)$$

BS K

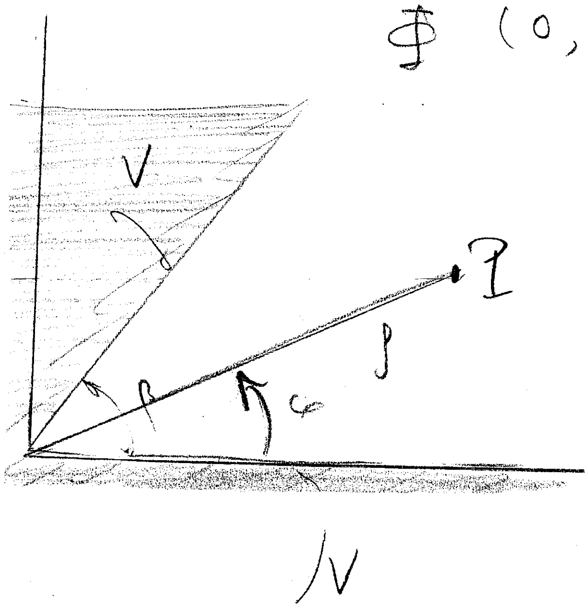
$$\Phi = \frac{2V}{\pi} \tan^{-1} \frac{2\,\mathrm{Im}\, z}{1 - |z|^2}$$

$$= \frac{2V}{\pi} \tan^{-1} \left\{ \frac{2\, e^{-\pi y/a} \sin \frac{\pi x}{a}}{\underbrace{1 - e^{-2\pi y/a}}_{e^{-\frac{\pi y}{a}} \left(e^{\pi y/a} - e^{-\pi y/a} \right)}} \right\}$$

$$= \frac{2V}{\pi} \tan^{-1} \left(\frac{\sin \frac{\pi x}{a}}{\sinh \frac{\pi y}{a}} \right) \qquad 2.65.$$

2.11. iki-boyutlu düzlemsel kutupsal koordinatlarda Laplace denk.i

Φ $(0, \beta)$ arasında



$$\nabla^2 \Phi = 0 \qquad \nabla^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\Phi(\rho, \varphi) = R(\rho)\, \Phi(\varphi)$$

$$\frac{1}{\rho}\Phi\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)+\frac{R}{\rho^2}\frac{d^2\Phi}{d\varphi^2}=0 \quad \times\frac{\rho^2}{R\Phi}$$

$$\underbrace{\frac{\rho}{R}\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)}_{\nu^2}+\underbrace{\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}}_{-\nu^2}=0$$

$$\left.\begin{aligned}\rho\frac{d}{d\rho}\left(\rho\frac{dR}{d\rho}\right)&=\nu^2R\\ \frac{d^2\Phi}{d\varphi^2}&=-\nu^2\Phi\end{aligned}\right\}\quad \nu\neq0 \text{ için } R=a\rho^{\nu}+b\rho^{-\nu} \text{ olduğundan}$$

$$\Phi=A\cos(\nu\varphi)+B\sin(\nu\varphi)$$

$$\rho^2R''+\rho R'-\nu^2R=0$$

$$R''+\frac{1}{\rho}R'-\frac{\nu^2}{\rho^2}R=0$$

$\nu=0$ için $R=a_0+b_0\ln\rho$

olduğundan

$$\Phi=A_0+B_0\varphi$$

$$\Phi(\rho,\varphi)=(a_0+b_0\ln\rho)(A_0+B_0\varphi)$$
$$+\left(a\rho^{\nu}+b\rho^{-\nu}\right)\left(A\cos(\nu\varphi)+B\sin(\nu\varphi)\right)$$

$\varphi=0$ da $\forall\rho\geq0$ için $\Phi=V$

$\varphi=\beta$ da $\forall\rho\geq0$ " $\Phi=V$

1) $\rho=0$ da Φ sonlu olmalı. Önce bunu yapalım.

1) $b=0$, $b_0=0$

$$\Phi(\rho,\varphi) = a_0 A_0 + a_0 B_0 \varphi + a\rho^{\nu}\left[A\cos(\nu\varphi)+B\sin(\nu\varphi)\right]$$

$$\Phi(\rho,0) = a_0 A_0 + a\rho^{\nu} A = V$$ olması için $A=0$ olmalı.

olmalı

$$\Rightarrow \Phi(\rho,\varphi) = V + a_0 B_0 \varphi + aB\rho^{\nu}\sin\nu\varphi$$

$\varphi=\beta$ da $\Phi(\rho,\beta)=V$ olmalı

$$= V + a_0 \underbrace{B_0}_{0}\beta + aB\rho^{\nu}\underbrace{\sin\nu\beta}_{0}$$

$\nu\beta = n\pi$

$$\Rightarrow \Phi(\rho,\varphi) = V + \sum_n a_n \rho^{n\pi/\beta}\sin\left(\frac{n\pi}{\beta}\varphi\right)$$

$\rho\to 0$ civarında çözümü inceleyelim.

$$\Phi = V + a_1\rho^{\pi/\beta}\sin\frac{\pi\varphi}{\beta} + \underbrace{a_2\left(\rho^{\pi/\beta}\right)^2\sin\frac{2\pi\varphi}{\beta}+\dots}_{\simeq 0}$$

$$\Phi = V + a_1\rho^{\pi/\beta}\sin\frac{\pi\varphi}{\beta}$$

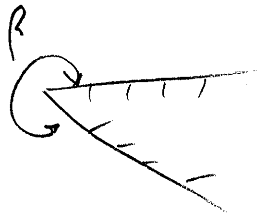
$$E_\rho = -\frac{\partial\Phi}{\partial\rho} = -a_1\frac{\pi}{\beta}\rho^{\frac{\pi}{\beta}-1}\sin\frac{\pi\varphi}{\beta}$$

$$E_\varphi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \varphi} = +a_1 \frac{\pi}{\beta}\rho^{\frac{\pi}{\beta}-1}\cos\frac{\pi\varphi}{\beta}$$

$\rho \sim 0$ civarında $E \sim \rho^{\frac{\pi}{\beta}-1}$ şeklinde davranır.

$$\left.\begin{array}{ll} \frac{\pi}{\beta}-1>0 \Rightarrow \rho\to 0 & (\beta<\pi) \\ \frac{\pi}{\beta}-1<0 \Rightarrow \rho\to 0 & (\beta>\pi) \end{array}\right\} \begin{array}{l} E\to 0 \\ E\to\infty \end{array}$$

$\beta = 2\pi \Rightarrow E \sim \rho^{-1/2}$



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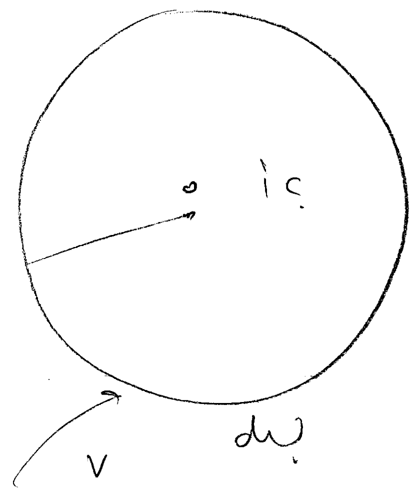
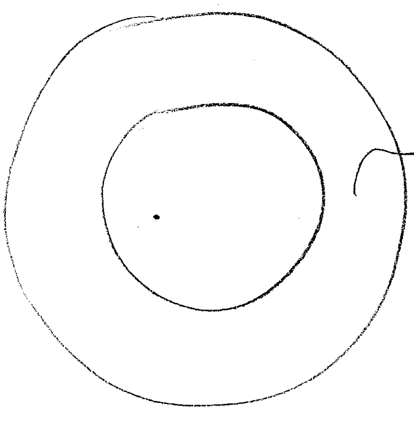


$\beta = 2\pi$

$\left.\begin{array}{l} b=0 \\ B_0=0 \\ b_0=0 \end{array}\right)$ iç $\qquad \left.\begin{array}{l} a=0 \\ B_0=0 \\ b_0=0 \end{array}\right)$ dış



ara bölgede

$B_0 = 0$

Ara bölge için genel çözüm,

$$\Phi(\rho,\varphi) = a_0 + b_0 \ln\rho + \sum_{n=1}^{\infty} a_n \rho^{n} \sin(n\varphi + \alpha_n)$$

$$+ \sum_{n=1}^{\infty} b_n \rho^{-n} \sin(n\varphi + \beta_n)$$

Çözüm bölgesi orijini kapsıyor ise $b_n = 0$

$$\left.\begin{matrix}\varphi = 0 \\ \varphi = \beta\end{matrix}\right) \rho \geq 0 \text{ için } \Phi = V \text{ olsun}$$

$$\Phi(\rho,\varphi) = V + \sum_{m=1}^{\infty} a_m \rho^{m\pi/\beta} \sin(m\pi\varphi/\beta)$$

potansiyel β da kendini yenilediğinden,

$$\sin\nu\varphi = 0 \qquad \nu\beta = m\pi$$

$\rho \sim 0$ yakınında ilk terim önemli olacak $\quad \Phi \sim V + a_1 \rho^{\pi/\beta} \sin \pi\varphi/\beta$

$$\vec{E} = -\vec{\nabla}\Phi$$

$$E_\rho = -\frac{\partial\Phi}{\partial\rho} = -\frac{\pi a_1}{\beta} \rho^{\frac{\pi}{\beta}-1} \sin \pi\varphi/\beta$$

$$E_\varphi = -\frac{1}{\rho}\frac{\partial\Phi}{\partial\varphi} = -\frac{\pi a_1}{\beta} \rho^{\pi/\beta - 1} \cos \pi\varphi/\beta$$

$$\sigma = -\frac{1}{4\pi r}\frac{\partial \Phi}{\partial \varphi} = \frac{E_\varphi}{4\pi}$$

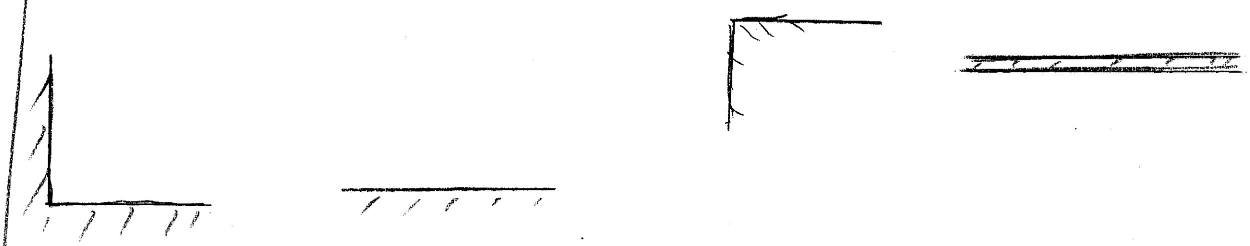
$$\sigma\Big|_{\varphi=0} = \frac{E_\varphi(\rho,0)}{4\pi} = -\frac{\pi a_1}{4\beta\pi}\,\rho^{\pi/\beta-1}$$

$$\sigma\Big|_{\varphi=\beta} = -\frac{\pi a_1}{4\pi\beta}\,\rho^{\pi/\beta-1}$$

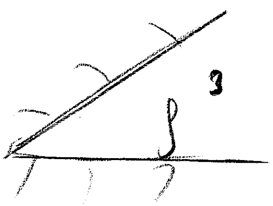
$\frac{\pi}{\beta}-1 > 0 \rightarrow \sigma$ küçük ρ zati küçük

$< 0 \rightarrow \sigma$ büyük

$\beta =$	$90°$	$180°$	$270°$	$360°$
$\sigma \sim$	ρ	$\sigma \sim$ sbt	$\sigma \sim \rho^{-1/3}$	$\rho^{-1/2}$

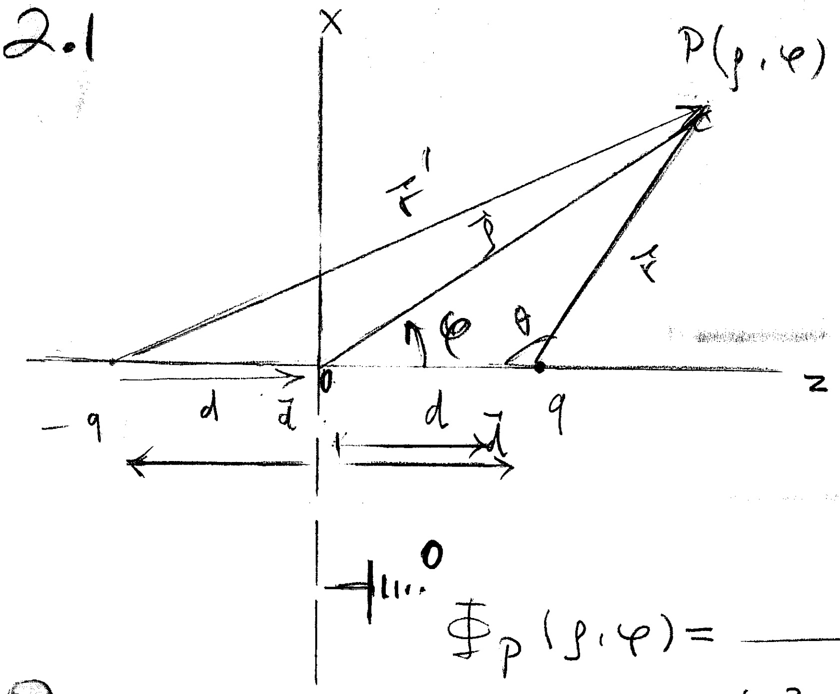


$45°$



(a)

2.1



kutupsal koordinatlarda,

$$\Phi_P(r,r') = \frac{q}{r} - \frac{q}{r'}$$

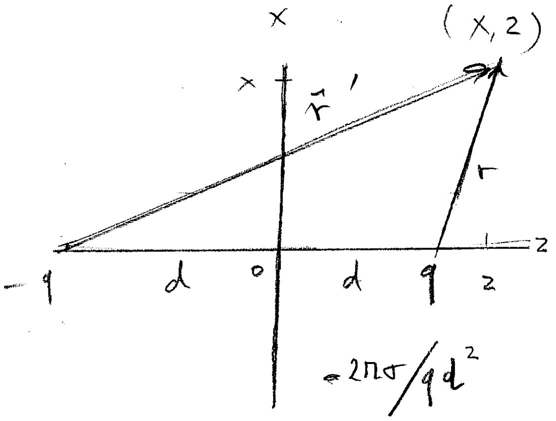
$$\vec{r} = \vec{\rho} - \vec{d}$$
$$\vec{r}\,' = \vec{\rho} + \vec{d}$$

$$\Phi_P(\rho,\varphi) = \frac{q}{(\rho^2 + d^2 - 2\rho d\cos\varphi)^{1/2}} - \frac{q}{(\rho^2 + d^2 + 2\rho d\cos\varphi)^{1/2}}$$

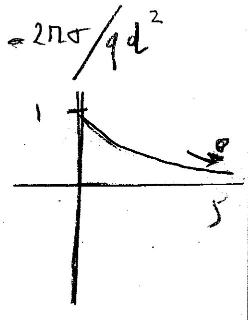
$$\sigma = \frac{-1}{4\pi}\frac{\partial \Phi}{\partial d}\bigg|_{\varphi=\pi/2} = \frac{-q}{4\pi}\left\{\frac{-\frac{1}{\cancel{2}}(-\cancel{2}\rho\cos\varphi + \cancel{2}d)}{(\rho^2+d^2-2\rho d\cos\varphi)^{3/2}} - \frac{-\frac{1}{\cancel{2}}(+\cancel{2}\rho\cos\varphi + \cancel{2}d)}{(\rho^2+d^2+2\rho d\cos\varphi)^{3/2}}\right\}_{\pi/2}$$

$$= -\frac{q}{2\cancel{4\pi}}\frac{\cancel{2}d}{(\rho^2+d^2)^{3/2}} = -\frac{qd}{2\pi r^3}$$

$$\sigma\,(\rho=0,\ r=d) = \frac{-q\cancel{d}}{2\pi d^{\cancel{3}}} = -\frac{q}{2\pi d^2}$$



$$\sigma\,(r=\infty) = 0$$

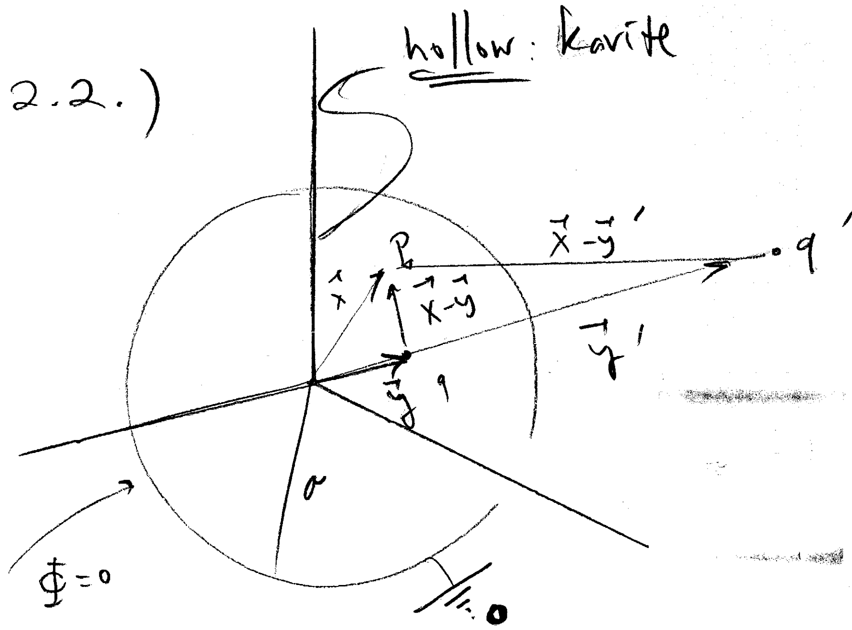


kartezyen koord.

$$\Phi(x,z) = \frac{q}{(x^2+y^2+(z-d)^2)^{1/2}} - \frac{q}{(x^2+y^2+(z+d)^2)^{1/2}}$$

$$\sigma(x,y) = -\frac{1}{4\pi}\frac{\partial\Phi}{\partial z}\bigg|_{z=0} = -\frac{q}{2\pi}\frac{d}{(x^2+y^2+d^2)^{3/2}}\ ;\ \rho^2 = x^2+y^2$$ yukarıdaki ile aynı

2.2.)



$$q' = -\frac{a}{y}q$$

not: $y<a$ olduğundan $|q'|>q$ dur.

$$y' = \frac{a^2}{y} \Rightarrow y' > y$$

(a) $$\Phi_P(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|}$$

(b) $$\sigma = -\frac{1}{4\pi}\frac{\partial \Phi}{\partial x}\Big|_{x=a} = -\frac{q}{4\pi a^2}\left(\frac{a}{y}\right)\frac{\left(1-a^2/y^2\right)}{\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\gamma\right)}$$

(c) $$F = \frac{qq'}{(y'-y)^2} = \frac{-q(a/y)q}{\left(\frac{a^2}{y}-y\right)^2} = -\frac{aq^2y}{(a^2-y^2)^2}$$ çekici

(d) (i) Küre sbt. bir pot. de tutulur ise (bkz. kes. 2.4)

$$\Phi_P(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|} + \frac{Va}{|\vec{x}|}$$

(ii) Küre yüzeyi üzerinde yalıtılmış bir Q yükü var ise (bkz. kes. 3.)

$$\Phi_P(\vec{x}) = \frac{q}{|\vec{x}-\vec{y}|} + \frac{q'}{|\vec{x}-\vec{y}'|} + \frac{Q-q'}{|\vec{x}|}$$

$$(x-x')^2+(y-y')^2=\rho^2+\rho'^2-2\rho\rho'(\cos\varphi\cos\varphi'-\sin\varphi\sin\varphi')$$

$$\Phi(\rho,\varphi,z)=\frac{1}{2\pi}\int_0^{2\pi}\int_0^{a}\underbrace{\Phi(\rho',\varphi',0)}_{V}d\varphi'\rho'd\rho'\frac{z}{[\rho^2+\rho'^2-2\rho\rho'\cos(\varphi-\varphi')+z^2]^{3/2}}$$

$\boxed{I}$

(c) Eksen boyunca $\rho=0$

$$\Phi(z)=\frac{Vz}{2\pi}\int_0^{2\pi}d\varphi'\int_0^{a}\frac{\rho'd\rho'}{[\rho'^2+z^2]^{3/2}}$$

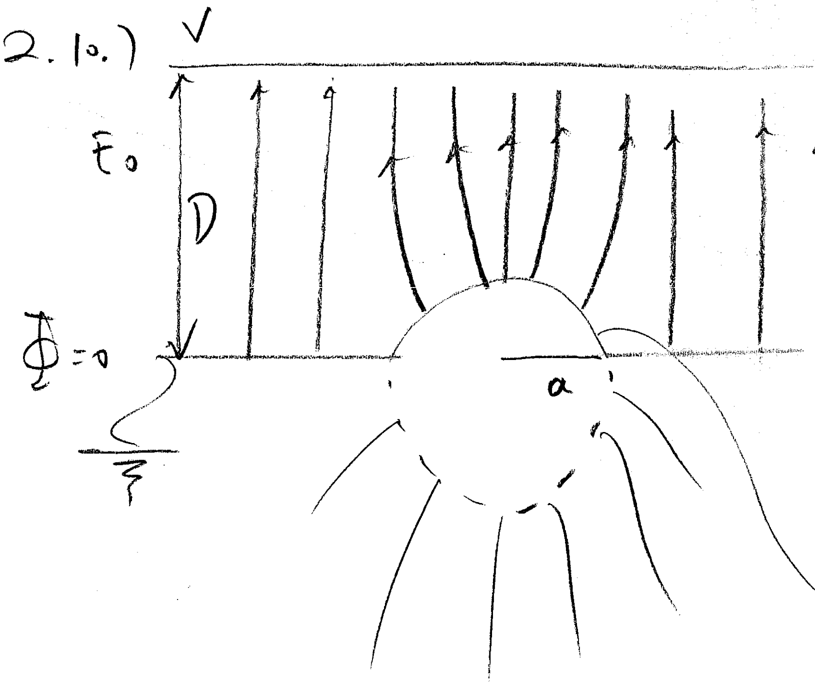
$$=\frac{Vz}{\cancel{2\pi}}\cancel{2\pi}\frac{1}{2}\int_0^{a}\frac{d\rho'^2}{(\rho'^2+z^2)^{3/2}}=V\left[1-\frac{z}{\sqrt{a^2+z^2}}\right]$$

(d) $\rho^2+z^2\gg a^2$ uzak bölge I'i binom serisine açar isek, binom..

$$\frac{1}{(\rho^2+z^2)^{3/2}}\left[1+\frac{\rho'^2-2\rho\rho'\cos(\varphi-\varphi')}{\rho^2+z^2}\right]^{-3/2}$$

$$=\frac{1}{(\rho^2+z^2)^{3/2}}\left[1-\frac{3}{2}\frac{\rho'^2-2\rho\rho'\cos(\varphi-\varphi')}{\rho^2+z^2}+\frac{3}{2}\frac{5}{2}\frac{1}{2}\times\left(\frac{\rho'^2-2\rho\rho'\cos(\varphi-\varphi')}{\rho^2+z^2}\right)^2+\dots\right]$$

2.10.)



Ara bölgede, potansiyel, düzgün $\vec{E}_0$ alanına konulmuş a yarıçaplı iletken küre ile aynı,

$$\Phi(r,\theta) = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta$$

boss (yumru)

$D \gg a$

$$\sigma_{\text{yarıküre}} = -\frac{1}{4\pi}\frac{\partial \Phi}{\partial r}\Big|_{r=a} = \frac{3}{4\pi}E_0\cos\theta$$

$$\sigma_{\text{düz}} = \frac{-1}{4\pi}\frac{\partial \Phi}{\partial z}\Big|_{z=0} = \frac{-E_0}{4\pi}\frac{\partial}{\partial z}\left[(x^2+y^2+z^2)^{1/2} - a^3(x^2+y^2+z^2)^{-1}\right]_{z=0}$$

$$= \frac{E_0}{4\pi}\left(1 - \frac{a^3}{(x^2+y^2)^{3/2}}\right)$$

(b)

$$Q_{\text{yarıküre}} = \frac{3E_0}{4\pi}\int \cos\theta \, da$$

$$= \frac{3E_0}{4\pi}a^2\int_0^{\pi/2}\int_0^{2\pi}\cos\theta\,\sin\theta\,d\theta\,d\varphi$$

$$= -\frac{3E_0 a^2}{4\pi}2\pi\int_1^0 \cos\theta\, d(\cos\theta)$$

BS K

$$= \frac{1}{2\pi}\int_0^{2\pi} \underbrace{\left\{1+2\sum_{n=1}^{\infty}\frac{\rho^n}{b^n}\cos n(\varphi-\varphi')\right\}}_{\#} \Phi(b,\varphi')\,d\varphi'$$

$$\# = 1+\sum\left[\frac{\rho^n}{b^n}e^^{in(\varphi-\varphi')}+\frac{\rho^n}{b^n}e^{-in(\varphi-\varphi')}\right]$$

geometrik serisi

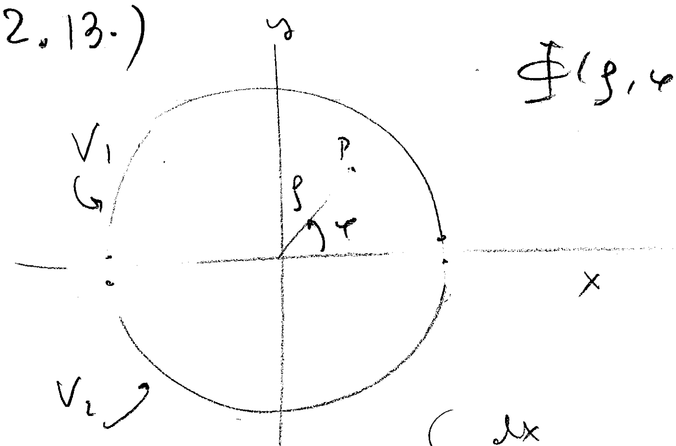
$$\sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$$

$$= 1+\left[\frac{(\rho/b)\,e^{i(\varphi-\varphi')}}{1-(\rho/b)\,e^{i(\varphi-\varphi')}}+\frac{(\rho/b)\,e^{-i(\varphi-\varphi')}}{1-(\rho/b)\,e^{i(\varphi-\varphi')}}\right] = \frac{1-(\rho/b)^2}{1-(\rho/b)\,e^{i(\varphi-\varphi')}-(\rho/b)\,e^{-i(\varphi-\varphi')}+(\rho/b)^2}$$

$$= \frac{1-(\rho/b)^2}{1-(2\rho/b)\cos(\varphi-\varphi')+(\rho/b)^2} = \frac{b^2-\rho^2}{b^2+\rho^2-2b\rho\cos(\varphi-\varphi')}$$

$$\Phi(\rho,\varphi) = \frac{1}{2\pi}\int_0^{2\pi}\frac{b^2-\rho^2}{b^2+\rho^2-2b\rho\cos(\varphi-\varphi')}\,\Phi(b,\varphi')\,d\varphi'$$

Prb. 2.13.)



$$\Phi(\rho,\varphi) = \frac{1}{2\pi}\left[\int_0^{\pi} V_1\frac{b^2-\rho^2}{b^2+\rho^2-2b\rho\cos(\varphi-\varphi')}\,d\varphi' + \int_{\pi}^{2\pi} V_2\frac{b^2-\rho^2}{b^2+\rho^2-2b\rho\cos(\varphi-\varphi')}\,d\varphi'\right]$$

$$\int\frac{dx}{a+b\cos x} = \frac{2}{\sqrt{a^2-b^2}}\,tg^{-1}\frac{(a-b)\tan x/2}{\sqrt{a^2-b^2}} \qquad a^2-b^2>0$$

$a = b^2+\rho^2$

$\varphi-\varphi' = x$

$b = 2b\rho$