BÖLÜM 12

RELATİVİSTİK PARÇACIKLARIN DİNAMİĞİ ve ELEKTROMAGNETİK ALANLAR

12.1. Dış EM alanda yüklü relativistik parçacık için Lagranjiyen ve Hamiltoniyen

$\vec{E}$ ve $\vec{B}$ dış alanlarındaki e yüklü bir parçacık için

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}\vec{u}\times\vec{B}$$

$$\frac{dE}{dt} = e\,\vec{u}\cdot\vec{E}$$

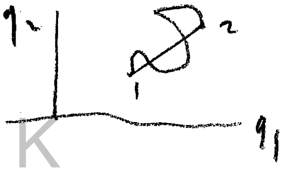
kovaryant formda $\dfrac{dU^\alpha}{d\tau} = \dfrac{e}{mc} F^{\alpha\beta} U_\beta$ (11.144)

biçiminde yazılabilir $U^\alpha = (\underset{\gamma c}{U_0}, \gamma\vec{u})$, $d\tau = \dfrac{dt}{\gamma}$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Relativistik olmayan incelemede $L = L(q_i(t), \dot{q}_i(t); t)$

$$A = \int L(q_i(t), \dot{q}_i(t); t)\,dt \quad \text{Eylem}$$



$\delta A = 0$ $\quad \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right) - \dfrac{\partial L}{\partial q_i} = 0$

(a) Relativistik lagranjiyene elementer yaklaşım.

$$A = \underbrace{\int_{\tau_1}^{\tau_2} \gamma L}_{\text{Lor. inv.}} \underbrace{d\tau}_{\text{Lor. invaryant}}$$

İlk önce L'nin LD özelliklerine bakalım. 1. postülaya göre eylem integrali Lor. skaleri olmalı.

$\Downarrow$

γL lor. inv. olmalı

* inversiyon altında invaryant olması için konuma bağlı olmaması gerek, sadece hıza bağlı ise

$$U^\alpha U_\alpha = c^2$$ (niceliği hızın fonksiyonu olan tek Lİ fonk. o zaman serbest parçacık lagranjiyeni $\propto \frac{1}{\gamma}$)

$$L \propto \left(1 - \frac{u^2}{c^2}\right)^{1/2}$$ serbest parçacık için

$$A = \alpha \int_1^2 ds \qquad ds^2 = c^2 dt^2 - d\vec{x}^2$$

$$= c^2 \left(1 - \frac{d\vec{x}^2}{c^2 dt^2}\right) dt^2$$

$$\Rightarrow A = \alpha c \int_1^2 dt \sqrt{1 - \frac{u^2}{c^2}}$$

küçük hızlar için $L^{NR} = \alpha c \left(1 - \frac{1}{2}\frac{u^2}{c^2} + \dots\right)$

$$= \frac{1}{2} m u^2$$

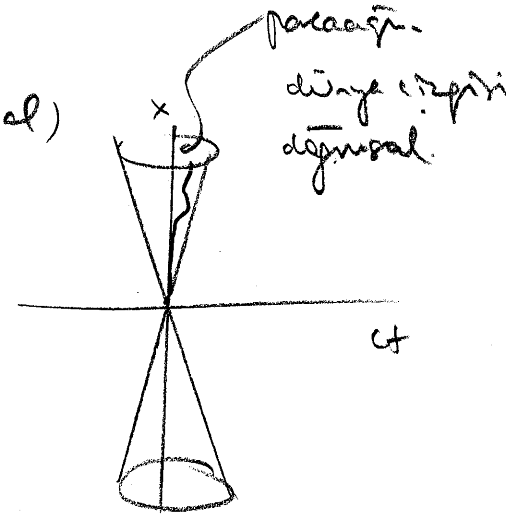
$$\Rightarrow \frac{-\alpha}{c} = m$$

$$\alpha = -mc$$

$$\boxed{L = -mc^2 \sqrt{1 - \frac{u^2}{c^2}}}$$

Euler-Lagrange denklemlerinden ,

$$\frac{d}{dt}(\gamma m\vec{u}) = \vec{0} \quad (\text{hareket doğrusal})$$



Klasik $\begin{cases} L = T - V \\ V = e\Phi \end{cases}$

relativistik $\longrightarrow$ $L^{R}_{int} \xrightarrow{v \ll c} L^{NR}_{int} = -e\Phi$

A^{α}'nın zaman bileşeni olduğundan γL_{int} şöyle olmalı

$$\Rightarrow \underbrace{\gamma L_{int}}\ \propto A^{\alpha} \times \underbrace{(\text{4'lü vektör})_{\alpha}}_{U_\alpha} \qquad A^{\alpha} = (\Phi, \vec{A})$$

(ölçekleme invaryanslığına sahip olduğundan hız/ları açık olarak içeremez) $\propto U_\alpha A^\alpha \qquad U_\alpha = (\gamma c, -\gamma\vec{u})$

$$\Downarrow$$

$$L_{int} = -\frac{e}{\gamma c} U_\alpha A^\alpha$$

$$= \frac{e}{c}\vec{A}\cdot\vec{u} - e\Phi$$

$$L^{R}_{T} = -mc^2\left(1 - \frac{u^2}{c^2}\right)^{1/2} + \frac{e}{c}\vec{A}\cdot\vec{u} - e\Phi$$ (yüklü bir parçacık için R Lagrangiyen)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial u_i}\right) - \frac{\partial L}{\partial x_i} = 0$$

$$p_i = \frac{\partial L}{\partial u_i} = -m\cancel{c^2}\,\frac{\frac{1}{\cancel{2}}(-\cancel{2}u_i)\frac{1}{\cancel{c^2}}}{(1-u^2/c^2)^{1/2}} + \frac{e}{c}A_i = \gamma_u m u_i + \frac{e}{c}A_i$$

$$\vec{P} = \underbrace{\gamma_u m \vec{u}}_{\text{basit lineer mom.}} + \frac{e}{c}\vec{A} \qquad (\vec{x} \text{ koord.na eşlenik kanonik mom.})$$

$$= \vec{p} + \frac{e}{c}\vec{A}$$

$H \to \vec{x}, \vec{P}$'nin fonk. udur. $\quad P_i = \frac{\partial \mathcal{L}}{\partial u_i}$

$$H = \vec{P}\cdot\vec{u} - L$$

$$\frac{d\vec{P}}{dt} = \frac{d\vec{p}}{dt} + \frac{e}{c}\frac{d\vec{A}(\vec{x})}{dt} = \frac{d\vec{p}}{dt} + \frac{e}{c}\frac{\partial\vec{A}}{\partial t} + \frac{e}{c}(\vec{u}\cdot\vec{\nabla})\vec{A}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}\cdot\vec{\nabla} \quad (\text{konvektif türev})$$

$$\frac{\partial \mathcal{L}}{\partial r_i} \to \vec{\nabla} L \qquad \vec{\nabla}(\vec{a}\cdot\vec{b}) = \cancel{(\vec{a}\cdot\vec{\nabla})\vec{b}} + (\vec{b}\cdot\vec{\nabla})\vec{a} + \vec{a}\times\cancel{(\vec{\nabla}\times\vec{b})} + \vec{b}\times(\vec{\nabla}\times\vec{a})$$

$$\begin{cases} \vec{a} \to \vec{A} \\ \vec{b} \to \vec{u} \end{cases}$$

$$\vec{\nabla} L = \frac{e}{c}\vec{\nabla}(\vec{A}\cdot\vec{u}) - e\vec{\nabla}\Phi$$

$$= \frac{e}{c}\left\{(\vec{u}\cdot\vec{\nabla})\vec{A} + \vec{u}\times(\vec{\nabla}\times\vec{A})\right\} - e\vec{\nabla}\Phi$$

$$\frac{d\vec{p}}{dt} + \frac{e}{c}\frac{\partial\vec{A}}{\partial t} + \frac{e}{c}(\vec{u}\cdot\vec{\nabla})\vec{A} = \frac{e}{c}(\vec{u}\cdot\vec{\nabla})\vec{A} + \underbrace{\frac{e}{c}\vec{u}\times\vec{B} - e\vec{\nabla}\Phi}$$

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\vec{u}\times\vec{B} + e\left(-\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right)$$

$$\boxed{\frac{d\vec{p}}{dt} = \frac{e}{c}\vec{u}\times\vec{B} + e\vec{E}}$$

BS K

$$\overbrace{q_i\,,\dot q_i}^{\text{Lineer bağımlılar}}\,,t \rightarrow L$$

$$\underbrace{q_i\,,p_i\,,t}_{\text{lineer bağımsızlar}} \rightarrow \mathcal{H}$$

$$\mathcal{H} = \sum_i p_i \dot q_i - L$$

$$= \vec p\cdot\vec u - L$$

$$\vec P = m\gamma\vec u + \frac{e}{c}\vec A$$

$$m\gamma\vec u = \vec P - \frac{e}{c}\vec A \Rightarrow m^2\gamma^2u^2 = \left(\vec P - \frac{e}{c}\vec A\right)^2$$

$$\Rightarrow m^2\frac{1}{\left(1-\frac{u^2}{c^2}\right)}\frac{u^2}{c^2} = \frac{1}{c^2}\left(\vec P - \frac{e}{c}\vec A\right)^2$$

$$= m^2(\gamma^2-1)$$

$$\Rightarrow m^2\gamma^2c^2 - m^2c^2 = \left(\vec P - \frac{e}{c}\vec A\right)^2$$

$$m\gamma c = \sqrt{\left(\vec P - \frac{e}{c}\vec A\right)^2 + m^2c^2}$$

$$\vec u = \frac{c\vec P - e\vec A}{m\gamma c} = \frac{c\vec P - e\vec A}{\sqrt{\left(\vec P - \frac{e}{c}\vec A\right)^2 + m^2c^2}}$$

$$\mathcal{H} = \vec P\cdot\frac{\left(c\vec P - e\vec A\right)}{\sqrt{\left(\vec P - \frac{e}{c}\vec A\right)^2 + m^2c^2}} + \underbrace{\frac{mc^2}{\gamma} - \frac{e}{c}\frac{\left(c\vec P - e\vec A\right)}{\sqrt{\left(\vec P - \frac{e}{c}\vec A\right)^2 + m^2c^2}}\cdot\vec A}_{-L_T^R} + e\Phi$$

$$= \sqrt{\left(c\vec P - e\vec A\right)^2 + m^2c^4} + e\Phi$$

→ Hamilton hareket denk.'ini bul. Lorentz K.'ni bul

Sistemin toplam enerjisi $\underbrace{(W - e\Phi)^2 - (c\vec{P} - e\vec{A})^2}_{\text{bu 4'lü skaler çarpım}} = (mc^2)^2$

$$P_\alpha P^\alpha = (mc)^2 \leftarrow \text{şeklinde,}$$

$$P^\alpha = \left(\frac{1}{c}(W - e\Phi) \, , \, \vec{P} - \frac{e}{c}\vec{A} \right)$$

(b) Kovaryant türetim

$\vec{x}$ ve $\vec{u}$ 'lar yerine 4'lü vektörler alınır.

$$L_{serbest} = -mc^2 \left(1 - u^2/c^2\right)^{1/2}$$

$U_\alpha U^\alpha = c^2$ idi

$$\gamma L_{serbest} = -mc\sqrt{u^\alpha u_\alpha}$$

$$\tilde{L}_S = -\frac{mc}{\gamma}\sqrt{U^\alpha U_\alpha}$$

$$A = -\int_{\tau_1}^{\tau_2} mc\sqrt{U_\alpha U^\alpha}\, d\tau$$

$U^\alpha U_\alpha = c^2$ (bağ koşulu)

hareket denk.i üzerine!

$$U^\alpha \frac{dU_\alpha}{d\tau} = 0$$

$$\sqrt{\frac{dx^\alpha}{d\tau}\frac{dx_\alpha}{d\tau}}\, d\tau = \sqrt{U^\alpha U_\alpha}\, d\tau$$

$$= \sqrt{g^{\alpha\beta}\frac{dx_\alpha}{d\tau}\frac{dx_\beta}{d\tau}}\, d\tau$$

$$A = -mc \int_{s_1}^{s_2} \sqrt{g^{\alpha\beta} \frac{dx_\alpha}{d\tau} \frac{dx_\beta}{d\tau}}\, d\tau$$

$$= -mc \int_{s_1}^{s_2} \underbrace{\sqrt{g^{\alpha\beta} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds}}\, ds}_{c\,d\tau}$$

$\delta A = 0$ geodezik eğrileri verir.

$$\delta A = -mc \int \delta \sqrt{g^{\alpha\beta} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds}}\, ds = 0$$

$$\int \frac{g^{\alpha\beta} \frac{dx_\alpha}{ds}\, \delta \frac{dx_\beta}{ds}\, ds}{\sqrt{}}$$

$$= \frac{g^{\alpha\beta} \frac{dx_\alpha}{ds}}{\sqrt{}}\, \delta x_\beta \Bigg|_1^2 - \int \frac{d}{ds}\left[\frac{g^{\alpha\beta} \frac{dx_\alpha}{ds}}{\sqrt{}} \right] \delta x_\beta$$

$$\frac{d}{ds}\left[\frac{g^{\alpha\beta} \frac{dx_\alpha}{ds}}{\sqrt{}} \right] = 0 \quad \text{EL denk. i}$$

<u>Serbest parçacık için:</u> $m \frac{d^2}{d\tau^2} x^\beta = 0$ beklendiği gibi! / etkileşim alanında $\frac{e}{c} U_\alpha A^\alpha \rightarrow \frac{e}{c} \frac{dx_\alpha}{d\tau} A^\alpha$ eklenmeli!

$$A = -\int_{s_1}^{s_2} \left[mc \sqrt{g^{\alpha\beta} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds}} + \frac{e}{c} \frac{dx_\alpha}{d\tau} A^\alpha \right] ds$$

$$\boxed{\frac{d}{ds}\left[\frac{\partial \tilde{L}}{\partial \left(\frac{dx_\alpha}{ds} \right)} \right] - \partial^\alpha \tilde{L} = 0}$$

$$\frac{\partial \tilde{L}}{\partial\left(\frac{dx_\alpha}{ds}\right)} = -mc \frac{g^{\alpha\beta}\frac{dx_\beta}{ds}}{\sqrt{g^{\alpha\beta}\frac{dx_\alpha}{ds}\frac{dx_\beta}{ds}}} - \frac{e}{c} A^\alpha$$

$$\partial^\alpha \tilde{L} = -\frac{e}{c}\frac{dx_\alpha}{ds}\frac{\partial A^\alpha}{\partial x_\alpha}$$

$$\frac{d}{ds}\left[\frac{mc\, g^{\alpha\beta}\frac{dx_\beta}{ds}}{\sqrt{g^{\alpha\beta}\frac{dx_\alpha}{ds}\frac{dx_\beta}{ds}}} + \frac{e}{c}A^\alpha\right] = \frac{e}{c}\frac{dx_\alpha}{ds}\frac{\partial A^\alpha}{\partial x_\alpha}$$

$$\frac{1}{\sqrt{}}\frac{d}{ds}\left[\quad\right] = \frac{e}{c}\frac{1}{\sqrt{}}\frac{dx_\alpha}{ds}\frac{\partial A^\alpha}{\partial x_\alpha} \qquad \sqrt{}\, ds = c\, d\tau$$

$$\frac{d}{d\tau}\left[m g^{\alpha\beta}\frac{dx_\beta}{d\tau} + \frac{e}{c}A^\alpha(x)\right] = \frac{e}{c}\frac{dx_\beta}{d\tau}\frac{\partial A^\beta}{\partial x_\alpha}$$

$$m\frac{d^2 x^\alpha}{d\tau^2} + \frac{e}{c}\frac{dx_\beta}{d\tau}\frac{\partial A^\alpha}{\partial x_\beta} = \frac{e}{c}\frac{dx_\beta}{d\tau}\frac{\partial A^\beta}{\partial x_\alpha}$$

$$\boxed{m\frac{d^2 x^\alpha}{d\tau^2} = \frac{e}{c}\frac{dx_\beta}{d\tau}\left(\partial^\alpha A^\beta - \partial^\beta A^\alpha\right)}$$

Conjugate mom.

$$P^\alpha = -\frac{\partial \tilde{L}}{\partial\left(\frac{dx_\alpha}{ds}\right)}$$

$$= mU^\alpha + \frac{e}{c}A^\alpha \quad \text{[illegible] mom.}$$

$$\tilde{H} = \frac{1}{2}\left(P_\alpha U^\alpha + \tilde{L}\right)$$

$$= \frac{1}{2m}\left(P^\alpha - \frac{e}{c}A^\alpha\right)\left(P_\alpha - \frac{e}{c}A_\alpha\right) - \frac{1}{2}mc^2$$

Hamilton hareket denklemleri $\begin{cases} \dfrac{dx^\alpha}{d\tau} = \dfrac{\partial \tilde{H}}{\partial p_\alpha} \\ \dfrac{dP^\alpha}{d\tau} = -\dfrac{\partial \tilde{H}}{\partial x_\alpha} \end{cases}$

$$\frac{dx^\alpha}{d\tau} = \frac{1}{m}\left(P^\alpha - \frac{e}{c}A^\alpha\right)$$

$P_\beta - \frac{e}{c}A_\beta$

$$\frac{dP^\alpha}{d\tau} = \frac{e}{mc}\, P^\beta \,\frac{\partial A_\beta}{\partial x_\alpha}$$

$\partial^\alpha A^\beta$

$\frac{d^2x^\alpha}{d\tau^2}$ yi bul.

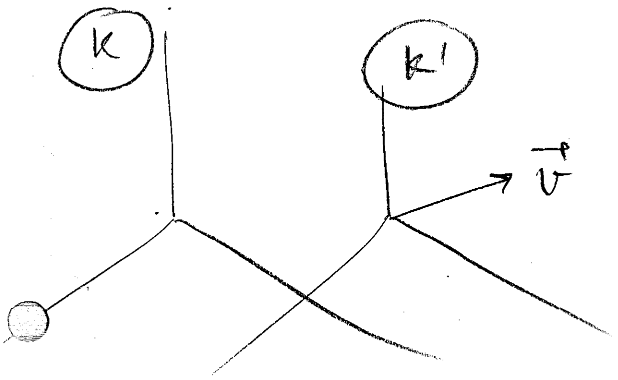
i) $\tilde{H}$ Lorentz skaleri, ancak her zaman enerji değil.

ii) $\tilde{H} \equiv 0$ $\quad U_\alpha U^\alpha = c^2$ ve $P^\alpha = -\dfrac{\partial \tilde{L}}{\partial\left(\frac{\partial x_\alpha}{ds}\right)} = mU^\alpha + \frac{e}{c}A^\alpha$

kullanılarak.

A.O. Barut Electrodynamics & Classical Theory of fields & particles (1964) Mc Millan p 68

12.2. Coulomb yasası ve Özel görelilikten Maxwell denk.nin, magnetik kuvvetin ve magnetik alanın elde edilmesi problemi



(p^α, t')

K': $\quad \dfrac{d\vec{p}\,'}{dt'} = \vec{F}' \qquad \vec{u}\,' = \dfrac{d\vec{x}\,'}{dt'}$

$$\left(\frac{d\vec{p}}{dt}\right)_{\parallel} = \left(\frac{d\vec{p}\,'}{dt'}\right)_{\parallel} + \frac{\gamma_{(\vec{v})}}{c^2}\left[\vec{u}\times\left(\vec{v}\times\frac{d\vec{p}\,'}{dt'}\right)\right]_{\parallel}$$

$$\left(\frac{d\vec{p}}{dt}\right)_{\perp} = \gamma_v\left(\frac{d\vec{p}\,'}{dt'}\right)_{\perp} + \frac{\gamma_{(\vec{v})}}{c^2}\left[\vec{v}\times\left(\vec{v}\times\frac{d\vec{p}\,'}{dt'}\right)\right]_{\perp}$$ çıkar.

K' de parçacık NR hareket ediyor olsun, ve $\vec{F}' = q\vec{E}\,'$ olsun.

K da,

$$\vec{B} = \frac{\gamma_v}{qc}\left(\vec{v}\times\frac{d\vec{p}\,'}{dt'}\right) = \frac{\gamma_v}{qc}\left(\vec{v}\times q\vec{E}\,'\right) = \frac{\gamma_v}{c}\left(\vec{v}\times\vec{E}\,'\right)$$

Lorentz kuvveti $\quad \dfrac{q}{c}\left(\vec{v}\times\vec{B}\right)$

K' de gerçek bir $\vec{B}\,'$ var $\Rightarrow$ ne olur?

$$\vec{F}' = q\vec{E}' + \frac{q}{c}\vec{u}' \times \vec{B}'$$

$u \approx 0$ alınır $\Rightarrow$ tartışma geçerli değil.

Lorentz skaler pot.:

$$L = -mc^2 \sqrt{1 - \frac{u^2}{c^2}}\; e^{g\phi(x)/mc^2}$$

g: bağlanma sbt.i

$$\tilde{L} = -mc \sqrt{g^{\alpha\beta} \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds}}\; e^{g\phi(x)/mc^2}$$

$$\frac{d}{ds}\left(\frac{\partial \tilde{L}}{\partial \left(\frac{dx_\alpha}{ds} \right)} \right) - \frac{\partial}{\partial x^\alpha} \tilde{L} = 0$$

Hareket denk.:

$$m \frac{dU^\alpha}{d\tau} = g\left(\partial^\alpha \phi - \frac{1}{c^2} U^\alpha \underbrace{U_\beta \partial^\beta \phi}\right)$$

(burada hız mag. alan ile etkileşim terimi gibi görünüyor, ancak mag. alan sadece $F^{\alpha\beta}$ de var.)

$$\frac{d\vec{p}}{dt} = -\frac{g}{\gamma_u} \vec{\nabla}\phi - \frac{g\vec{p}}{mc^2}\left(\frac{\partial \phi}{\partial t} + \vec{u} \cdot \vec{\nabla}\phi \right)$$

12.3. Üniform sbt. bir mag. alan içindeki bir parçacığın hareketi

paralel olmayan üniform, statik $\vec{E}$ ve $\vec{B}$ alanları içerisindeki yüklü parçacığın hareketi. $\vec{E} \perp \vec{B}$ önce incelenecek.

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\,\vec{v}\times\vec{B} \qquad \frac{dE}{dt} = 0$$

$= e\,\vec{v}\cdot\vec{E} \neq 0 \Rightarrow E$ zamanla sbt değil.

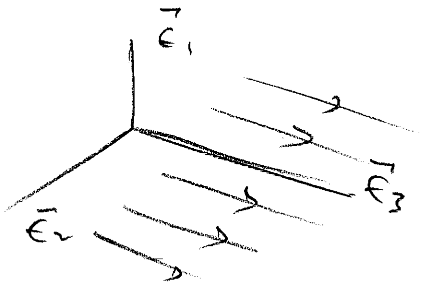
(parçacık mag. alandan enerji kazanmıyor)

$$\vec{v} = \frac{c^2}{E}\,\vec{p} \qquad \frac{E}{c^{\cancel{2}}}\frac{d\vec{v}}{dt} = \frac{e}{\cancel{c}}\,\vec{v}\times\vec{B}$$

$$\left\{ \begin{array}{l} \dfrac{ec}{E}\,\vec{B} = \vec{\omega}_B \\ \qquad = \dfrac{e}{\gamma m c}\,\vec{B} \\ \text{presesyon frekansı} \end{array} \right.$$

$$\frac{d\vec{v}}{dt} = \frac{ec}{E}\,\vec{v}\times\vec{B}$$

$$= \vec{v}\times\vec{\omega}_B \quad \rightarrow \text{bu çözülecek.}$$



$$\vec{v} = v_1\vec{\epsilon}_1 + v_2\vec{\epsilon}_2 + v_3\vec{\epsilon}_3$$

$$\longrightarrow \vec{B} = B\,\vec{\epsilon}_3 \Rightarrow \vec{\omega}_B = \omega_B\,\vec{\epsilon}_3$$

$$\vec{v}\times\vec{\omega}_B = v_2\,\omega_B\,\vec{\epsilon}_1 - v_1\,\omega_B\,\vec{\epsilon}_2$$

$$\boxed{\frac{dv_1}{dt} = v_2\,\omega_B \qquad \frac{dv_2}{dt} = -v_1\,\omega_B \qquad \frac{dv_3}{dt} = 0}$$

$$\frac{d}{dt}(v_1 + i v_2) = -i(v_1 + i v_2)\,\omega_B$$

$$\frac{dv_\perp}{dt} = -i\,v_\perp\,\omega_B \Rightarrow v_\perp = v_0\,e^{-i\omega_B t}$$

$$\overbrace{v_1 + i v_2}^{v_\perp} = v_0 \left(\cos \omega_B t - i \sin \omega_B t \right)$$

$$v_\perp(0) = v_0$$

$$\begin{cases} v_1(t) = v_0 \cos \omega_B t \\ v_2(t) = -v_0 \sin \omega_B t \end{cases}$$

$$\vec{v}(t) = v_1 \vec{\epsilon}_1 + v_2 \vec{\epsilon}_2 + v_\parallel \vec{\epsilon}_3$$

$$= v_\parallel \vec{\epsilon}_3 + v_0 \left(\cos \omega_B t \, \vec{\epsilon}_1 - \sin \omega_B t \, \vec{\epsilon}_2 \right)$$

$$= v_\parallel \vec{\epsilon}_3 + v_0 \left(\frac{e^{i\omega_B t} + e^{-i\omega_B t}}{2} \vec{\epsilon}_1 + i \frac{e^{i\omega_B t} - e^{-i\omega_B t}}{2} \vec{\epsilon}_2 \right)$$

$$= v_\parallel \vec{\epsilon}_3 + v_0 \left[\frac{e^{i\omega_B t}}{2} \left(\vec{\epsilon}_1 + i \vec{\epsilon}_2 \right) + \frac{e^{-i\omega_B t}}{2} \left(\vec{\epsilon}_1 - i \vec{\epsilon}_2 \right) \right]$$

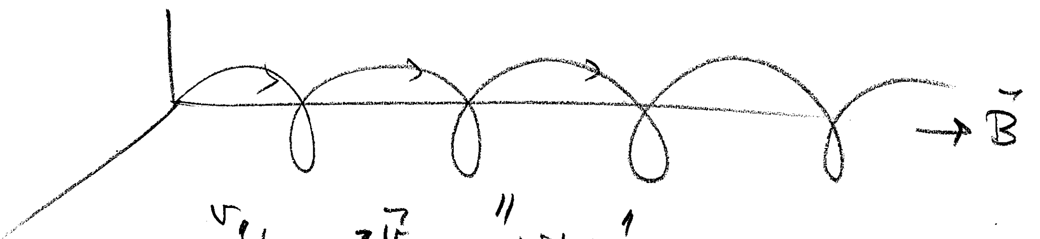
$v_\perp(0) = v_0 = \omega_B a$ presesyon yarıçapı

$$\vec{v}(t) = v_\parallel \vec{\epsilon}_3 + \omega_B a \, \mathrm{Re} \left[e^{-i\omega_B t} \left(\vec{\epsilon}_1 - i \vec{\epsilon}_2 \right) \right]$$

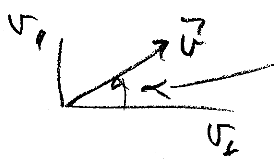
$$\frac{dx(t)}{dt} = v(t)$$

jirasyon merkezinin konumu.

$$\vec{x}(t) = \vec{x}_0 + v_\parallel t \, \vec{\epsilon}_3 + i a \, e^{-i\omega_B t} \left(\vec{\epsilon}_1 - i \vec{\epsilon}_2 \right)$$



$a > a$



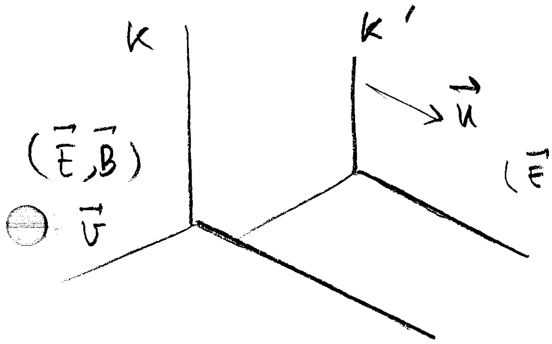
"pitch" açısı (meyil mertebe)

$$\alpha = tg^{-1} \frac{v_\parallel}{\omega_B a}$$

12.4. $\vec{E}$ ve $\vec{B}$ içinde hareket

(i) $\vec{E} \perp \vec{B}$

$$\frac{d\vec{p}}{dt} = e\left[\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right] \qquad \frac{dE}{dt} = e\,\vec{v}\cdot\vec{E}$$ (zaman içinde E sbt değil.)



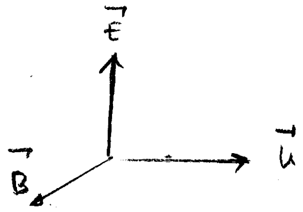
$$\frac{d\vec{p}'}{dt} = e\left(\vec{E}' + \frac{1}{c}\vec{v}'\times\vec{B}'\right) \quad \text{LI}$$

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta}\times\vec{B}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E})$$

$$\vec{B}' = \gamma(\vec{B} - \vec{\beta}\times\vec{E}) - \frac{\gamma^2}{\gamma^2+1}\vec{\beta}(\vec{\beta}\cdot\vec{E})$$

$$\vec{\beta} = \frac{\vec{u}}{c}$$ ($\vec{E}$ ve $\vec{B}$ için $\vec{\beta} \to -\vec{\beta}$ konulur.)

$|\vec{E}| < |\vec{B}|$ olsun!



$$\vec{u} = c\,\frac{\vec{E}\times\vec{B}}{B^2}$$ seçersek

$$|\vec{u}| = c\,\frac{E}{B}$$

$$\gamma_u = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} = \left(1 - \frac{E^2}{B^2}\right)^{-1/2}$$

$\vec{E}'_{\parallel} = \vec{B}'_{\parallel} = 0$ olacaktır.

$$\vec{E}'_{\perp} = \gamma\left(\vec{E} + \frac{1}{B^2}(\vec{E}\times\vec{B})\times\vec{B}\right) \qquad -\vec{B}\times(\vec{E}\times\vec{B}) = -[B^2\vec{E} - \underbrace{\cancel{(\vec{B}\cdot\vec{E})}}_{0}\vec{B}] = -B^2\vec{E}$$

$$\vec{B}'_{\perp} = \gamma\left(\vec{B} - \frac{1}{B^2}(\vec{E}\times\vec{B})\cancel{\times\vec{E}}\right)$$

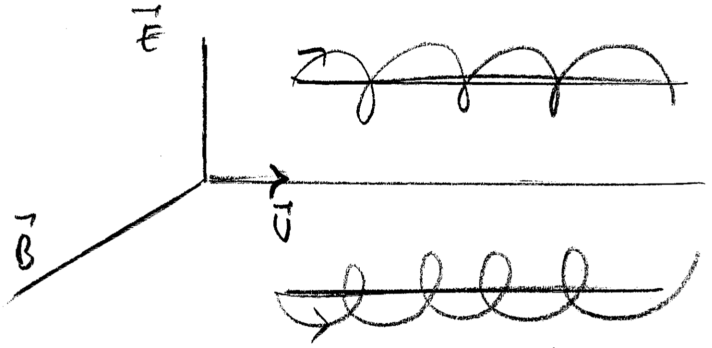
$$\vec{E}_\perp{}' = \gamma(\vec{E} - \vec{E}) = 0$$

$$\vec{B}_\perp{}' = \gamma(\vec{B} - \vec{E}) = \gamma \vec{B}\left(1 - \frac{E^2}{B^2}\right)$$

$$\boxed{\vec{B}_\perp{}' = \frac{1}{\gamma}\vec{B}}$$

$$\frac{d\vec{p}\,'}{dt} = e\left(\cancel{\vec{E}} + \frac{1}{c}\vec{v}\,' \times \vec{B}\,'\right)$$

çözümü istendiğinde bir önceki probleme indirgenir!



$\vec{E} \times \vec{B}$ sürüklenmesi

$|\vec{E}| > |\vec{B}|$ olsun

$$\vec{u}\,' = c\,\frac{\vec{E} \times \vec{B}}{B^2} \qquad \vec{E}_\parallel{}'' = \vec{B}_\parallel{}'' = 0 \quad \text{aynı şekilde.}$$

$$\vec{E}_\perp{}'' = \frac{1}{\gamma'}\vec{E} = \left(\frac{E^2 - B^2}{E^2}\right)^{1/2}\vec{E}$$

$$\vec{B}_\perp{}'' = \gamma'\left(\vec{B} - \frac{\vec{v}\,' \times \vec{E}\,'}{c}\right) = 0$$

K'' deki hareket sadece $\vec{E}_\perp{}''$ alan içindeki gibidir. (Hiperbolik bir hareket)

BS K

(ii) $\vec{E}$'nin $\vec{B}$ yönünde bileşeni varsa,

$\vec{E}\cdot\vec{B}=0$ $\quad B^2-E^2$ LI

Sağlanırsa

$|\vec{B}|>|\vec{E}| \Rightarrow \vec{E}=\vec{0} \quad \vec{B}\neq 0$

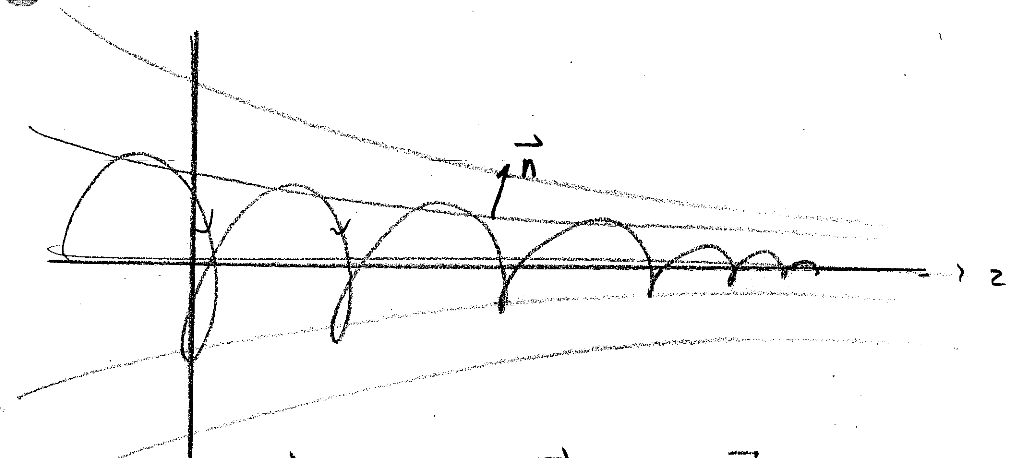
$|\vec{E}|<|\vec{B}| \Rightarrow \vec{B}=\vec{0} \quad \vec{E}\neq 0$

$\vec{E}\cdot\vec{B}\neq 0 \Rightarrow$ tüm Lorentz çerçevelerinde $\vec{E}$ ve $\vec{B}$ vardır.

12.5. Üniform olmayan statik magnetik alan içerisindeki Hareket:

astrofizikte termonükleer uygulamalarda.

Magnetik alan içindeki gradyent değişimi a'dan büyük, o zaman B yeterince "gentle". $\vec{B}$'nin büyüklüğünün uzaysal değişimi a'ya kıyasla büyük. $\vec{n}\cdot\vec{B}=0$ olsun.



B'nin değişimi a'nınkinden daha uzun bölgeye yayılmış.

$$\vec{n}\cdot\vec{B}=0 \quad , \vec{\omega}_B=\frac{e}{\gamma mc}\vec{B}(\vec{x})$$

$$\cong \frac{e}{\gamma mc}\left(B_0+\vec{n}\left(\frac{\partial \vec{B}}{\partial \xi}\right)_0 \xi+\cdots\right)$$

normal doğrultudaki değişim

$$\vec{\omega}_B \simeq \vec{\omega}_0 \left(1 + \frac{1}{B_0}\left(\frac{\partial \vec{B}}{\partial \xi}\right)_0 \vec{n}.\vec{x} + \dots\right)$$

Enine hareketteki değişimleri bulalım.

Varsayım: $\vec{V}_\perp = \vec{V}_0 + \vec{V}_1$ alan varlığı getirilen düzeltme. (küçük!)

alan düzgün iken enine hız

$$\frac{d\vec{v}_\perp}{dt} = \vec{v}_\perp \times \vec{w}_B$$

$$\frac{d\vec{v}_1}{dt} \simeq (\vec{v}_0 + \vec{v}_1) \times \vec{w}_0 \left[1 + \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \vec{n}.\vec{x}\right]$$

$$= \left\{(\vec{v}_0 + \vec{v}_1)\left[1 + \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \vec{n}.\vec{x}\right]\right\} \times \vec{w}_0$$

$$\simeq \left[\vec{v}_1 + \vec{v}_0 (\vec{n}.\vec{x}) \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0\right] \times \vec{w}_0$$

$$\vec{v}(t) = v_\parallel \vec{\epsilon}_3 + w_B a (\vec{\epsilon}_1 - i\vec{\epsilon}_2) e^{-i w_B t} \quad (2.4)$$

$$\vec{x}(t) = \left(\text{aynı } \vec{X}_0 \text{ atılıyor}\right) + v_\parallel t \vec{\epsilon}_3 + ia (\vec{\epsilon}_1 - i\vec{\epsilon}_2) e^{-i w_B t} \quad (2.4)$$

$\vec{v}_0$ ile $\vec{x}_0$ arasındaki ilişki

$$-\vec{w}_0 \times \vec{x}_0 = -v_\parallel t \, \cancel{\vec{w}_0 \times \vec{\epsilon}_3} - ia \vec{w}_0 \times (\vec{\epsilon}_1 - i\vec{\epsilon}_2) e^{-i w_B t}$$

$\vec{w}_0 \parallel \vec{\epsilon}_3$

$$= -ia w_0 (\vec{\epsilon}_2 + i\vec{\epsilon}_1) e^{-i w_B t}$$

$$= \underbrace{a w_0 (\vec{\epsilon}_1 - i\vec{\epsilon}_2) e^{-i w_B t}}$$

$$-\vec{w}_0 \times \vec{x}_0 = \vec{v}_0 \Rightarrow \vec{x}_0 = \frac{1}{w_0} (\vec{w}_0 \times \vec{x})$$ aşağıda! →

$$-\vec{\omega}_0 \times (\vec{\omega}_0 \times \vec{X}_0) = \vec{\omega}_0 \times \vec{v}_0$$

$$= -\left[(\vec{\omega}_0 \cdot \vec{X}_0)\vec{\omega}_0 - \omega_0^2 \vec{X}_0 \right] = \underline{\underline{\vec{\omega}_0 \times \vec{v}_0}}$$

$$\vec{X}_0 = \frac{1}{\omega_0^2}(\vec{\omega}_0 \times \vec{x}_0)$$ 'den buluyoruz.

$$+ \vec{v}_0 (\vec{n}\cdot\vec{x}) \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \times \vec{\omega}_0$$

$$\frac{d\vec{v}_1}{dt} = \left[\vec{v}_1 - \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \underbrace{\vec{\omega}_0 \times \vec{X}_0}_{-\vec{v}_0} (\vec{n}\cdot\vec{X}_0) \right] \times \vec{\omega}_0$$

● $\langle \vec{v}_1 \rangle = \vec{v}_G$ "sürüklenme hızı."

$$= \frac{1}{B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \vec{\omega}_0 \times \langle \vec{X}_{0\perp} (\vec{n}\cdot\vec{X}_0) \rangle$$

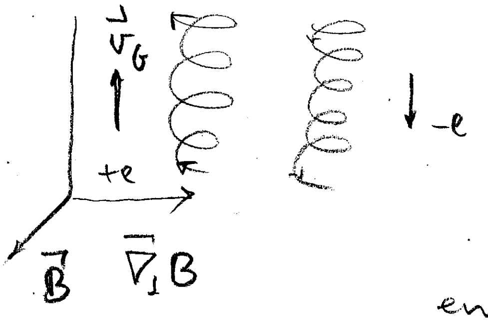
* $(X_0)_\perp$'nın $\vec{n}$ doğultusundaki bileşeni ortalamada sıfırdan farklı bir değer verir.

$$\omega = \frac{e}{\gamma c m}\vec{B}$$

● $\langle (\vec{X}_0)_\perp (\vec{n}\cdot\vec{X}_0) \rangle = \vec{n}\frac{a^2}{2}$ (X_0^2 ninki gibi)

$$\vec{v}_G = \frac{a^2}{2B_0}\left(\frac{\partial B}{\partial \xi}\right)_0 \vec{\omega}_0 \times \vec{n} = \frac{a^2}{2}\frac{1}{B_0}\frac{eB_0}{\gamma mc}(\vec{B}\times\vec{\nabla}_\perp B)$$

$$\frac{\vec{v}_G}{\omega_B a} = \frac{a}{2B^2}(\vec{B}\times\vec{\nabla}_\perp B)$$



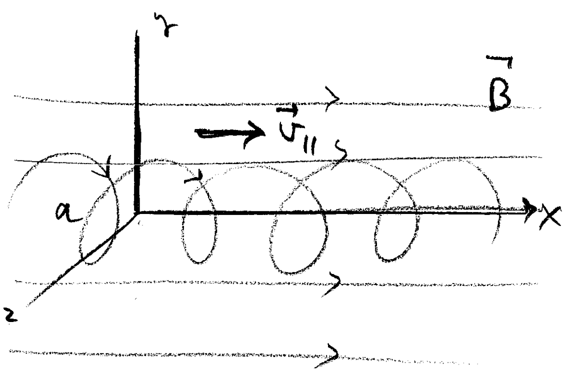
$$\left| \frac{\vec{\nabla}_\perp B}{B} \right| \ll 1 \quad \text{Gentle}$$

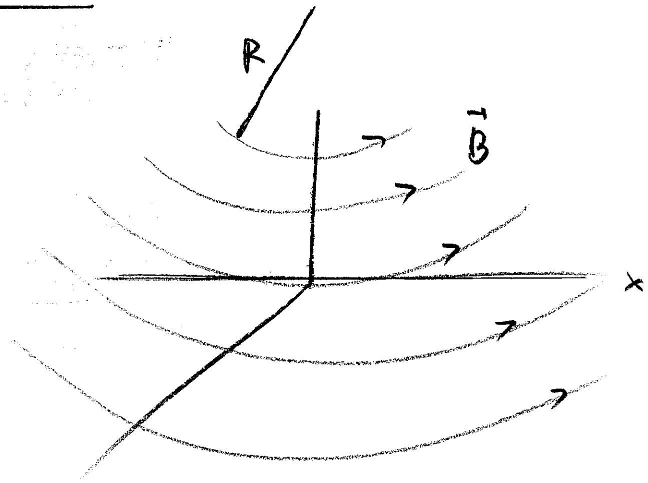
$$\frac{v_G}{\omega_B a} \ll 1$$

enine hıza nazaran sürüklenme hızı daha küçük!

BS K

(ii) Kuvvet çizgileri eğri ise;





$\frac{v_\parallel^2}{R}$ mertebesinde bir merkezkaç ivme etkisinde demektir.

$\gamma m \frac{\vec R}{R}\frac{v_\parallel^2}{R} = e\vec E_{eff}$ ~> bu kuvveti etkin $\vec E_{eff}$ alanından türemiş kabul edebiliriz.

$$\boxed{\vec E_{eff} = \frac{\gamma m}{e}\frac{v_\parallel^2}{R^2}\vec R}$$

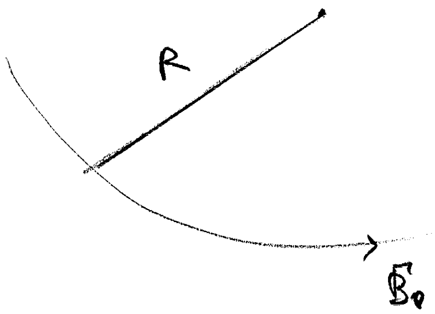
$\vec B_0$, $\vec E_{eff}$ $\quad |\vec E_{eff}| < |\vec B|$ 'a karşı gelecek çünkü $\vec a_m$ küçük.

$$\vec v_c = c\frac{\gamma m}{e} v_\parallel^2 \frac{\vec R\times\vec B_0}{R^2 B_0^2}$$ "curvature drift"

$$\vec u = c\frac{\vec E\times\vec B}{B}$$ $\vec E_{eff}$ $\vec B_0$

$$\vec V_c = \frac{v_\parallel^2}{\omega_B R}\left(\frac{\vec R\times\vec B_0}{R B_0}\right)$$ +e için
−e " $\quad \vec\omega_B \to -\vec\omega_B$

Lorentz kuvvet ifadesi kullanılarak aynısı elde edilebilir.



silindirik koordinatlarda çözüm yapacağız.

$$\boxed{\frac{d\vec{p}}{dt} = \frac{e}{c}\left(\vec{v}\times\vec{B}\right)} \qquad \vec{B} = (0, B_\phi, 0) = B_0\hat{\phi}$$

$$\vec{v} = \frac{c^2}{E}\vec{p}$$

$$\frac{E}{c^2}\frac{d\vec{v}}{dt} = \frac{e}{c}\left(\vec{v}\times\vec{B}\right) \qquad \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$$

$$\left.\frac{d\hat{\rho}}{dt} = \dot{\phi}\hat{\phi}, \quad \frac{d\hat{\phi}}{dt} = -\dot{\phi}\hat{\rho}\right\} \quad \frac{dv}{dt} = \ddot{\rho}\hat{\rho} + 2\dot{\rho}\dot{\phi}\hat{\phi} + \rho\ddot{\phi}\hat{\phi} - \rho\dot{\phi}^2\hat{\rho} + \ddot{z}\hat{z}$$

$$= \frac{ec}{E}\left(-\dot{z}B_\rho\hat{\rho} + \dot{\rho}B\hat{z}\right)$$

$$\underbrace{(\dot{\rho}, \rho\dot{\phi}, \dot{z})\wedge(0, B_\phi, 0)}$$

$$\frac{ceB}{E} = \omega_B$$

$$\begin{cases} \ddot{\rho} - \rho\dot{\phi}^2 = -\dot{z}\,\omega_B\dfrac{R}{\rho} \\ \rho\ddot{\phi} + 2\dot{\rho}\dot{\phi} = 0 \sim \rho^2\dot{\phi} = R v_{\parallel} \\ \ddot{z} = \omega_B\dot{\rho}\dfrac{R}{\rho} \qquad \dot{z} = \omega_B \ln(\rho/R) + v_0 \end{cases}$$

$R \gg a \Rightarrow$ 1. mertebeden yaklaşıklıkta $\dot{\phi} = \dfrac{v_{\parallel}}{R}$

$\ddot{\rho}$ ihmal edilirse (1. derece yaklaşıklıkta) $\dot{z} \cong \dfrac{v_{\parallel}^2}{\omega_B R}$ eğrilikten dolayı

sürüklenme

$$\vec{v}_c = \frac{v_{\parallel}^2}{\omega_B R}\left(\vec{R}\times\vec{B}_0\right) \text{ ile karşılaştırılabilir}$$

+ bir de gradyent varsa, çözüm bulunabilir, yani, $\vec{V}_B$ ve $\vec{V}_c$ birleştirilir.

çünkü eğrilikten dolayı $\vec{\nabla}\times\vec{B}=\vec{0}$

$$\Rightarrow \frac{\partial}{\partial\rho}(\rho B_\phi) - \cancel{\frac{\partial B_\rho}{\partial\phi}} = 0$$

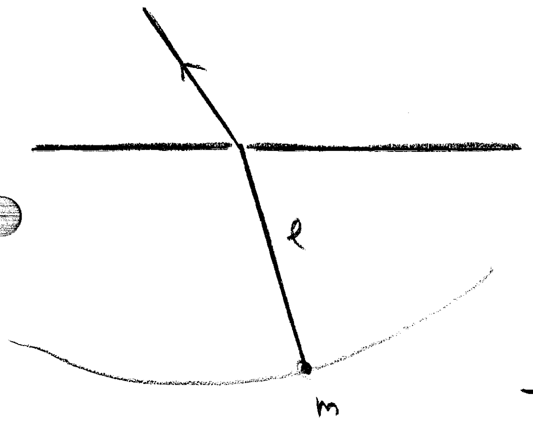
$$B + R\frac{\partial B}{\partial R} = 0$$

$$\frac{1}{B}\frac{\partial B}{\partial R} = -\frac{1}{R} \Rightarrow \frac{\vec{\nabla}_\perp \vec{B}}{B} = -\frac{\vec{R}}{R^2}$$

top. sürüklenme hızı $\vec{V}_D = \vec{V}_G + \vec{V}_c = \frac{1}{\omega_B R}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)\frac{\vec{R}\times\vec{B}}{RB}$ bul.

12.6. Adyabatik Değişmezlik

Şimdiye kadarki tartışmalarda hareket mag. kuvvet çizgilerine dik idi. Şimdi paralel olana bakalım.

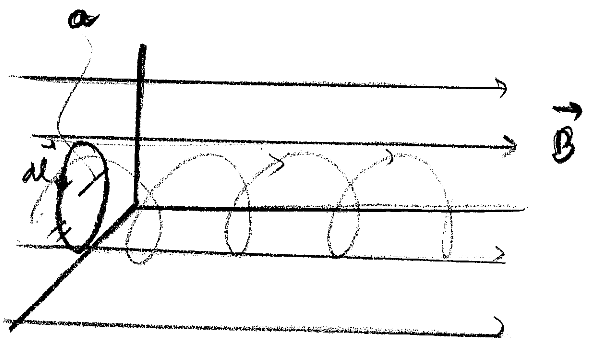


Sarkacın boyu periyoduna kıyasla yavaşça değiştirilirse eylem sabittir.
(sistem değişmiş olabilir)

$$J = \oint p_i\, dq_i$$

$\frac{E}{\nu} = sbt$ (eylem)

★ $\vec{B}$ alanındaki yüklü parçacığın hareketinin enine bileşeni periyodik.



$$J = \oint \vec{P}_\perp \cdot d\vec{l}$$

enine mom.

$$\vec{P} = \vec{p} + \frac{e}{c}\vec{A} \qquad 12.14$$

$$\vec{p} = \gamma m \vec{v}$$

$$J = \int m\gamma \vec{v}_\perp \cdot d\vec{l} + \frac{e}{c}\oint_c \vec{A}\cdot d\vec{l} \qquad \vec{v}_\perp \parallel d\vec{l}$$

$$= \int m\gamma v_\perp dl + \frac{e}{c}\oint_c \vec{A}\cdot d\vec{l}$$

$$= 2\pi m\gamma \omega_B a^2 + \frac{e}{c}\int_s (\vec{\nabla}\times\vec{A})\cdot\hat{n}\, da$$

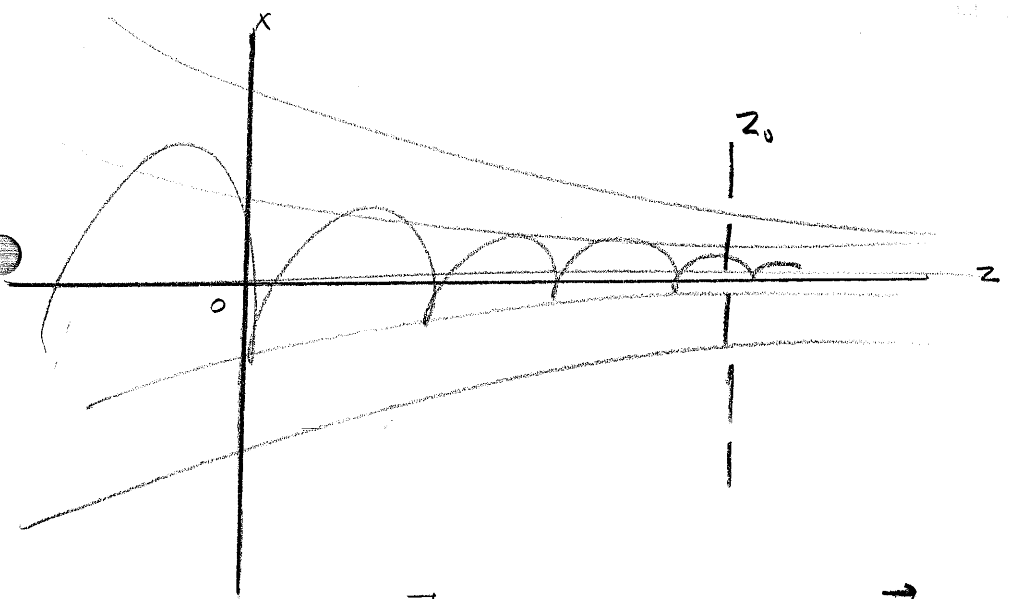
$$= 2\pi m\gamma \omega_B a^2 - \frac{e}{c}\int B\, da \qquad \hat{n} \uparrow\downarrow \vec{B}$$

$$= 2\pi m\gamma \omega_B a^2 - \frac{e}{c}\pi a^2 B \qquad \omega_B = eB/\gamma mc$$

$$= \frac{e}{c}\pi a^2 B = sbt = \gamma m \omega_B \pi a^2 = \frac{e}{c}(B\pi a^2)$$ parçacığın yörüngesinden geçen akı.

adyabatik değişmezler: $a^2 B$, $P_\perp^2/B$, $\gamma\mu$ magnetik mom.

$$\mu = e\frac{\omega_B}{2\pi}\frac{\pi a^2}{c} = \frac{1}{2c} e a^2 \omega_B$$ → akım halkasının mag. mom.i



→ z doğrultusunda $\vec{B}$ + x doğrultusunda $\vec{\nabla}B$ + kuvvet çizgilerinin eğriliği

$z=0$ da $v_0^2 = v_{0\perp}^2 + v_{0\parallel}^2$ B_0 düzgün

herhangi bir z için ne yazabiliriz?

$V_0^2 = V_{0\perp}^2 + V_{0\parallel}^2 = V_\perp^2 + V_\parallel^2$ (toplamı sbt)

$$\frac{P_\perp^2}{B} \rightarrow \frac{V_{0\perp}^2}{B} = \frac{V_\perp^2}{B(z)}$$ (adyabatik değişmezlikten.)

$$V_\parallel^2 = V_0^2 - V_\perp^2 = V_0^2 - \frac{V_{0\perp}^2}{B_0} B(z)$$

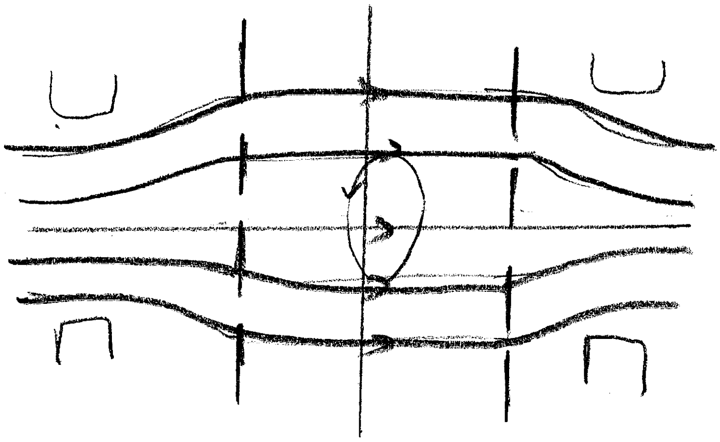
z boyunca hareket için ilk integrali verir.

$$V(z) = \frac{1}{2} m \frac{B(z)}{B_0} V_{\perp 0}^2$$

potansiyelindeki tek boyutlu hareket gibi.

$z = z_0$ için $V_\parallel = 0$ olur ($\frac{B(z)}{B_0}$ büyütülüp V_0^2'ye ulaşıldığında)

bu 'manyetik ayna' olarak adlandırılır.



$V_\parallel > 0$ olmasını istiyorsak $V_0^2 - V_{\perp 0}^2 \frac{B(z)}{B_0} \geq 0$ olmasını gerektirir.

$$V_{\perp 0}^2 + V_{\parallel 0}^2 - V_{\perp 0}^2 \frac{B(z)}{B_0} \geq 0$$

$$V_{\parallel 0}^2 > V_{\perp 0}^2 \left(\frac{B(z)}{B_0} - 1 \right)$$

$$\left(\frac{V_{\parallel 0}}{V_{\perp 0}} \right) > \left(\frac{B(z)}{B_0} - 1 \right)^{1/2}$$ bunları Lorentz denklemlerinden 1. yaklaşıklıkta bulabiliriz.

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho B_\rho)+\frac{1}{\rho}\cancel{\frac{\partial B_\phi}{\partial\phi}}+\frac{\partial B_z}{\partial z}=0$$

$$\frac{\partial}{\partial\rho}(\rho B_\rho)=-\rho\frac{\partial B_z}{\partial z}\qquad \rho B_\rho=-\int\rho\frac{\partial B_z}{\partial z}d\rho$$

bir peryod boyunca B düzgün ise

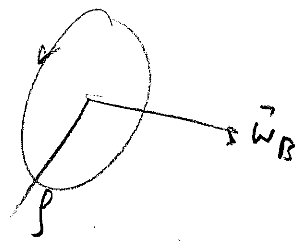
$$=-\frac{\partial B}{\partial z}\int\rho\, d\rho=-\frac{1}{2}\rho^2\frac{\partial B}{\partial z}$$

Lorentz kuvvetinin z-doğrultusundaki bileşeni,

$$\ddot{z}\cong\frac{e}{\gamma mc}\left(-\rho\dot{\phi}B_\rho\right)\qquad \dot{\phi}=\omega_B\quad z \text{ etrafındaki dairesel hareketin hızı}$$

olarak bulunmuştuk.

$$\ddot{z}=\frac{e}{2\gamma mc}\rho^2\dot{\phi}\frac{\partial B}{\partial z}\Rightarrow\rho^2\dot{\phi}=-a^2\omega_B$$



$$\ddot{z}=-\frac{e}{2\gamma mc}\underbrace{(a^2\omega_B)}_{v_{\perp 0}^2/\omega_B}\frac{\partial B}{\partial z}=-\frac{e}{2\gamma mc}\frac{v_{\perp 0}^2}{\omega_B}\frac{\partial B}{\partial z}$$

$$=-\frac{v_{\perp 0}^2}{B_0}\frac{\partial B}{\partial z}$$

bunu integre edilirse $V_\parallel$ bulunur.

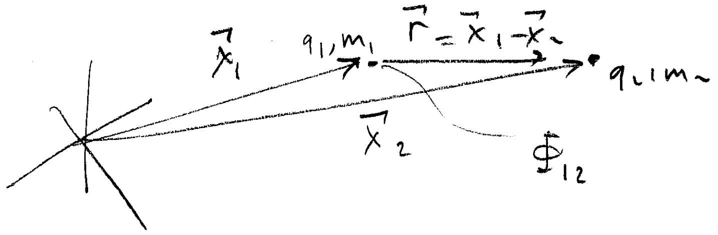
12.7. Darwin Lagranjiyeni (1920)

$m_1, m_2 \;/\; q_1, q_2 \;/\; \vec{x}_1, \vec{x}_2$

gibi iki parçacık alalım. $\vec{r} = \vec{x}_1 - \vec{x}_2$ bağıl koordinatlar

$$L^{NR}_{int} = -\frac{q_1 q_2}{r} \quad \left(= -q_1 \Phi_{12} \right)$$

Hareketli durumda A_{12} ye de gerek var.



$\vec{A}_{12}$ 'yi hesap ederken gecikme etkileri ihmal edilir ve $\frac{v}{c}$ mertebesine kadar alınır.

$$\frac{q_1}{c} \vec{v}_1 \cdot \underbrace{\vec{A}_{12}}_{v/c} \rightsquigarrow \sim \left(\frac{v}{c} \right)^2 \text{ düzeltmesi}$$

$\vec{\nabla} \cdot \vec{A} = 0$ Coulomb ayarı

$$\vec{A}_{12}(\vec{x}) \stackrel{\sim}{=} \frac{1}{c} \int \frac{\vec{J}_t(\vec{x})}{|\vec{x} - \vec{x}'|} d^3 x'$$

$$\vec{J}(\vec{x}') = q_2 \vec{v}_2 \, \delta(\vec{x}' - \vec{x}) = \vec{J}_\ell + \vec{J}_t$$

$$\vec{J}_t = \vec{J} - \vec{J}_\ell$$

$$\vec{J}_\ell = -\frac{1}{4\pi} \vec{\nabla} \int \frac{\vec{\nabla}' \cdot \vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3 x'$$

$$= -\frac{1}{4\pi} \vec{\nabla} \cdot \int \frac{\vec{J}(\vec{x}') \cdot (\vec{x} - \vec{x}') \, d^3 x'}{|\vec{x} - \vec{x}'|^3}$$

$$\vec J_\ell = \frac{1}{4\pi}\vec\nabla \int \frac{q_2 \vec v_2\, \delta(\vec x'-\vec x_2)\cdot(\vec x-\vec x')}{|\vec x-\vec x'|^3}\, d^3x'$$

$$= \frac{q_2}{4\pi}\vec\nabla\left(\frac{\vec v_2\cdot(\vec x-\vec x_2)}{|\vec x-\vec x_2|^3}\right)$$

$$\vec J_t(\vec x') = q_2\vec v_2\,\delta(\vec x'-\vec x_2) - \frac{q_2}{4\pi}\vec\nabla'\left(\frac{\vec v_2\cdot(\vec x'-\vec x_2)}{|\vec x'-\vec x|^3}\right)$$ (6.24 – 6.48 den.)

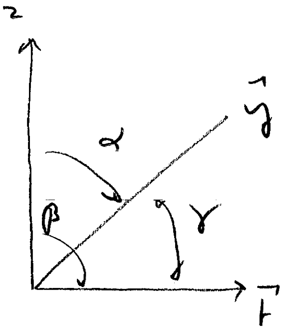
$$\vec A_{12}(\vec x) \cong \frac{q_2}{c}\vec v_2\frac{1}{r} - \frac{q_2}{4\pi c}\int \frac{1}{|\vec x-\vec x'|}\vec\nabla'\left(\frac{\vec v_2\cdot(\vec x'-\vec x_2)}{|\vec x'-\vec x|^3}\right)d^3\vec x'$$

$\vec x'-\vec x_2=\vec y$, $\vec x_1-\vec x' = \vec x_1-\vec x_2-\vec y=\vec r-\vec y$ $\quad \vec\nabla'\to\vec\nabla_{\vec y}$

$$\vec A_{12}(\vec x) = \frac{q_2}{c}\frac{\vec v_2}{r} - \frac{q_2}{4\pi c}\int\frac{1}{|\vec r-\vec y|}\vec\nabla_{\vec y}\left(\frac{\vec v_2\cdot\vec y}{y^3}\right)d^3\vec y$$

$$- \frac{q_2}{4\pi c}\int\left(\vec\nabla_{\vec y}\frac{1}{|\vec y-\vec r|}\right)\frac{\vec v_2\cdot\vec y}{y^2}\,d^3y$$

$$+\frac{q_2}{4\pi c}\vec\nabla_{\vec r}\int\frac{1}{|\vec y-\vec r|}\frac{\vec v_2\cdot\vec y}{y^2}\,d^3y .$$



$$\cos\gamma = \cos\alpha\cos\beta + \sin\alpha\sin\beta\cos(\alpha-\beta)$$

$$\int \frac{v_2\cos\alpha\; y}{y^3}\,\frac{y^2\,dy\,\sin\alpha\,d\alpha\,d\phi}{\sqrt{y^2+r^2-2yr\cos\gamma}}$$

$$\vec A_{12}(\vec x) = \frac{q_2\vec v_2}{cr} - \frac{q_2}{2c}\vec\nabla_r\left(\frac{\vec v_2\cdot\vec r}{r}\right)$$

$$= \frac{q_2}{2cr}\left[\vec v_2 + \vec r\,\frac{(\vec v_2\cdot\vec r)}{r}\right]$$

$$L_{int} = \frac{q_1 q_2}{r}\left\{-1+\frac{1}{2c^2}\left[\vec v_1\cdot\vec v_2 + \frac{(\vec v_1\cdot\vec r)(\vec v_2\cdot\vec r)}{r}\right]\right\}$$

İki elektronlu atomlarda "Bright etkileşmesi" (1930) (Relativistik düzeltme) olarak kullanılır. Çok sayıda yüklü parçacık varsa,

$$L_{int} = \frac{1}{2}\sum_i m_i \dot{r}_i^2 + \frac{1}{8c^2}\sum_i m_i \dot{r}_i^4 - \frac{1}{2}\sum_{ij} \frac{q_i q_j}{r_{ij}}$$

$$+ \frac{1}{4c^2}\sum_{ij} \frac{q_i q_j}{r_{ij}}\left[v_i v_j + \frac{(v_i \hat{r}_{ij}).(v_j \hat{r}_{ij})}{r_{ij}} \right]$$

$$\hat{r}_{ij} = \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

12.8 Elektromagnetik alan için Lagrange fonksiyonu

$$L = \sum_i L_i (q_i, \dot{q}_i ; t)$$

Kesikli ortamdan sürekli ortama geçeceğiz.

$$i = 1, 2, \ldots, n \longrightarrow x^k \quad : k = 1, \ldots, n$$

$$q_i \longrightarrow \phi_k(x) \text{ sürekli alan}$$

$$\dot{q}_i \longrightarrow \partial^\beta \phi_k(x)$$

$$L \longrightarrow \int \mathcal{L}(\phi_k, \partial^\beta \phi_k)\, d^4x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \longrightarrow \partial^\beta \frac{\partial \mathcal{L}}{\partial(\partial^\alpha \phi_k)} - \frac{\partial \mathcal{L}}{\partial \phi_k} = 0$$

Elektromagnetik Alan için:

"koordinatlar" $\longrightarrow A^\alpha$

" hız " $\longrightarrow \partial^\beta A^\alpha$

$$A = \int \mathcal{L}(\phi_k, \partial^\beta \phi_k)\, d^4x$$

Lor. inv. — orantı — Lor. skaler olmalı — Lor. inv.

$\mathcal{L} \propto (\text{hız})^2$, $F_{\alpha\beta}F^{\alpha\beta}$, $\underbrace{\tilde{F}_{\alpha\beta}F^{\alpha\beta}}_{\text{pseudoscalar}}$

$\mathcal{L} \propto F_{\alpha\beta}F^{\alpha\beta}$, $J^\alpha A_\alpha$ / parçacık!

$$= -\frac{1}{16\pi}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{c}J_\alpha A^\alpha \qquad F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha)$$

Euler-Lagrange denk. ini kullanabilmek için

$$\mathcal{L} = -\frac{1}{16\pi} g_{\lambda\mu} g_{\nu\sigma}(\partial^\mu A^\sigma - \partial^\sigma A^\mu)(\partial^\lambda A^\nu - \partial^\nu A^\lambda) - \frac{1}{c}J_\alpha A^\alpha$$

formunda yazarız.

$$\frac{\partial \mathcal{L}}{\partial(\partial^\beta A^\alpha)} = \frac{-1}{16\pi} g_{\lambda\mu} g_{\nu\sigma}\Big[\underbrace{\frac{\partial \partial^\mu A^\sigma}{\partial(\partial^\beta A^\alpha)}}_{(\delta^\mu_\beta \delta^\sigma_\alpha)} F^{\lambda\nu} - \delta^\sigma_\beta \delta^\mu_\alpha F^{\lambda\nu} + \delta^\lambda_\beta \delta^\sigma_\alpha F^{\mu\sigma} - \delta^\nu_\beta \delta^\lambda_\alpha F^{\mu\sigma}\Big]$$

$$= \frac{-1}{4\pi}F_{\beta\alpha} = \frac{1}{4\pi}F_{\alpha\beta}$$

$$\frac{\partial \mathcal{L}}{\partial A^\alpha} = \frac{-1}{c}J_\alpha \qquad \partial^\beta F_{\beta\alpha} = \frac{4\pi}{c}J_\alpha \quad \text{Inhom. Maxwell Denk.}$$

Hom. yap! $\partial^\alpha \tilde{\mathcal{F}}^{\alpha\beta} = 0$

12.10. Kanonik ve Simetrik Gerilim tensörleri

Korunum yasaları.

a. Genelleştirilmiş Hamiltoniyen.

kanonik mom. $p_i = \frac{\partial L}{\partial \dot{q}_i}$ $\quad H = \sum p_i \dot{q}_i - L$ } parçacık mekaniğinde

kanonik mom. ordinatı ile bağıntısı ile Hamilton fonksiyonunu yazdık.

$\frac{\partial L}{\partial t} = 0 \;\Rightarrow\; \frac{dH}{dt} = 0$

olmak şartı ile } olduğu gösterilebilir.

Alanlar için

$$H = \int \mathcal{H} \, d^3\vec{x} \qquad \mathcal{H} = \sum_k \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \phi_k}{\partial t}\right)} \frac{\partial \phi_k}{\partial t} - \mathcal{L}$$

mom.

(2. derece bir tensörün zaman bileşeni olmalı.)

4 [illegible] zaman bileşeni

$\phi_k(x)$ ve $\partial^\alpha \phi_k(x)$ alan değişkenlerinin bir fonksiyonu

$k = 1, 2, \ldots, n$

$\mathcal{H}$'nın kovaryant formu kanonik stres (gerilim) tensörü $T^{\alpha\beta}$

bu tensör

$$T^{\alpha\beta} = \sum_k \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi_k)} \partial^\beta \phi_k - g^{\alpha\beta} \mathcal{L}$$

Hamiltonyen yoğunluğunun kovaryant genellemesi olduğunu söyler.

$\phi_k \to A^\lambda \qquad \mathcal{L} = \frac{-1}{8\pi} F_{\mu\nu} F^{\mu\nu}$

$$T^{\alpha\beta} = \frac{\partial \mathcal{L}}{\partial(\partial_\alpha A^\lambda)} \partial^\beta A^\lambda - g^{\alpha\beta} \mathcal{L}$$

$$\overbrace{\frac{\partial \mathcal{L}}{\partial(\partial^\alpha A^\beta)}}^{12.87} = \frac{-1}{4\pi} F_{\alpha\beta} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$T^{\alpha\beta} = \frac{-1}{4\pi} \underbrace{g^{\cancel{\alpha\beta}\,\mu} F_{\mu\lambda}}_{\text{yanlış!}} \partial^\beta A^\lambda - g^{\alpha\beta} \mathcal{L}$$

$g = \mathrm{diag}\,(+---)$ $\qquad \mathcal{L} = \frac{1}{8\pi}\,(E^2 - B^2)$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$T^{00} = -\frac{1}{4\pi} F_{0\lambda}\,\partial^0 A^\lambda - \frac{1}{8\pi}\,(E^2 - B^2)$$

$$= \frac{-1}{4\pi}\left[F_{00}\,\partial^0 A^0 + F_{01}\,\partial^0 A^1 + F_{02}\,\partial^0 A^2 + F_{03}\,\partial^0 A^3 \right] - \frac{1}{8\pi}\,(E^2 - B^2)$$

$$= \frac{-1}{4\pi}\left[E_x \frac{1}{c}\frac{\partial A^1}{\partial t} + E_y \frac{1}{c}\frac{\partial A^2}{\partial t} + E_z \frac{1}{c}\frac{\partial A^3}{\partial t} \right] - \frac{1}{8\pi}\,(E^2 - B^2)$$

$$= \frac{-1}{4\pi}\,\vec{E}\cdot\frac{\partial \vec{A}}{\partial t} - \frac{1}{8\pi}\,(E^2 - B^2)$$

$$= \frac{1}{4\pi}\,\vec{E}\cdot(\vec{\nabla}\Phi + \vec{E}) - \frac{1}{8\pi}\,(E^2 - B^2) \qquad \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$= \frac{1}{8\pi}\,(\vec{E}^2 + \vec{B}^2) + \frac{1}{4\pi}\,\vec{E}\cdot\vec{\nabla}\Phi$$

$$= \frac{1}{8\pi}\,(\vec{E}^2 + \vec{B}^2) + \frac{1}{4\pi}\,\vec{\nabla}\cdot(\vec{E}\Phi) \qquad \vec{\nabla}\cdot\vec{E} = 0$$

$$T^{0i} = \frac{-1}{4\pi}\, g^{0\mu} F_{\mu\lambda}\,\partial^i A^\lambda = \frac{-1}{4\pi}\, g^{00} F_{0\lambda}\,\partial^i A^\lambda$$

$$= \frac{-1}{4\pi}\left(F_{01}\,\partial^i A^1 + F_{02}\,\partial^i A^2 + F_{03}\,\partial^i A^3 \right)$$

$$= \frac{+1}{4\pi}\left(E_x\,\partial^i A_x + E_y\,\partial^i A_y + E_z\,\partial^i A_z \right)$$

$$= \frac{1}{4\pi}\,\vec{E}\cdot\frac{\partial \vec{A}}{\partial x_i}$$

$$T^{01} = \frac{1}{4\pi} \vec{E} \cdot \frac{\partial \vec{A}}{\partial x}$$

$$= \frac{1}{4\pi} \Big[E_x \frac{\partial A_x}{\partial x} + E_y \frac{\partial A_y}{\partial x} + E_z \frac{\partial A_z}{\partial x}$$

$$+ E_y \frac{\partial A_x}{\partial y} - E_y \frac{\partial A_x}{\partial y}$$

$$+ E_z \frac{\partial A_x}{\partial z} - E_z \frac{\partial A_x}{\partial z} \Big]$$

$$= \frac{1}{4\pi} \left(\vec{E} \cdot \vec{\nabla} A_x + E_y B_z - E_z B_y \right)$$

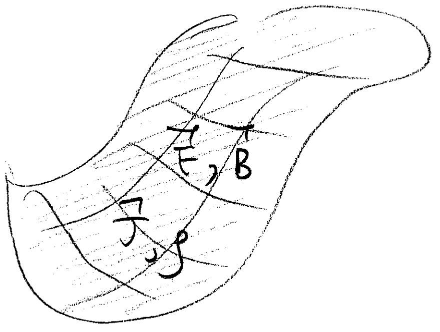
$$= \frac{1}{4\pi} \left(\vec{E} \cdot \vec{\nabla} A_x + (\vec{E} \times \vec{B})_x \right)$$

$$= \frac{1}{4\pi} \vec{\nabla} \cdot (\vec{E} A_x) + \frac{1}{4\pi} (\vec{E} \times \vec{B})_x \qquad \vec{\nabla} \cdot \vec{E} = 0$$

$$T^{0i} = \frac{1}{4\pi} \vec{\nabla} \cdot (\vec{E} A_i) + \frac{1}{4\pi} (\vec{E} \times \vec{B})_i \neq T^{i0}$$

$$\int T^{00} d^3x = \int \frac{E^2 + B^2}{8\pi} d^3x = \epsilon_f$$ (yükleri lokalize olduğunda)

$$\int T^{0i} d^3x = \frac{1}{4\pi} \int (\vec{E} \times \vec{B})_i \, d^3x = c \, P^i_f$$



$$\frac{\partial u}{\partial t} + \vec{\nabla} \cdot \vec{S} = - \vec{J} \cdot \vec{E}$$ Poynting teoremi

$$\Downarrow$$

$$\partial_\alpha T^{\alpha\beta} = 0$$ korunum yasası

$\partial_\alpha T^{\alpha\beta} = 0$ ı ispat edelim

Zincir kuralı:

$$\partial_\alpha T^{\alpha\beta} = \sum_k \left[\partial_\alpha \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi_k)} \partial^\beta \phi_k + \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi_k)} \partial_\alpha \partial^\beta \phi_k \right] - \partial^\beta \mathcal{L}$$

ı göz önüne alınır.

ise hareket denklemi

$$\partial_\alpha \frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi_k)} = \frac{\partial \mathcal{L}}{\partial \phi_k} \quad \text{idi}$$

$$\partial_\alpha T^{\alpha\beta} = \partial^\beta \mathcal{L} - \partial^\beta \mathcal{L} = 0 \quad ✓$$

$$\int \partial_\alpha T^{\alpha\beta} d^3x = 0 = \int \partial_0 T^{0\beta} d^3x + \int \partial_i T^{i\beta} d^3x = 0$$

$\beta = 0$ ise
$$\int \partial_0 T^{00} d^3x = \frac{1}{c} \int \frac{\partial}{\partial t} T^{00} d^3x = \frac{1}{c}\frac{d}{dt} \underbrace{\int T^{00} d^3x}_{\text{T.f. toplam enerji}}$$

$$\boxed{\frac{dE_f}{dt} = 0}$$

$\beta = i$ ise
$$\int \partial_0 T^{0i} d^3x = 0 \Rightarrow \boxed{\frac{d\vec{P}_f}{dt} = 0}$$

(b) Simetrik Gerilim Tensörü

Alan için açısal mom. tanımlanır.

$$\vec{L}_{alan} = \frac{1}{4\pi c} \int \vec{x} \wedge (\vec{E} \wedge \vec{B}) d^3x$$

$$M^{\alpha\beta\gamma} = T^{\alpha\beta} x^\gamma - T^{\alpha\gamma} x^\beta$$

korunumlu

$$\partial_\alpha M^{\alpha\beta\gamma} = 0 \qquad \frac{d\vec{L}}{dt} = 0$$

↳ için $T^{\alpha\beta}$ sim. olmalı.

$$\int \partial_\alpha M^{\alpha\beta\gamma} d^3x = \int \partial_0 M^{0\beta\gamma} d^3x + \int \partial_i M^{i\beta\gamma} d^3x$$

$$\Rightarrow \partial_\alpha M^{\alpha\beta\gamma} = \cancel{\partial_\alpha T^{\alpha\beta} x^\gamma} + \cancel{T^{\gamma\beta}} - \cancel{T^{\beta\gamma}} - \cancel{\partial_\alpha T^{\alpha\gamma}}$$

$$\boxed{T^{\alpha\beta} = T^{\beta\alpha}}$$

$T^{\alpha}_{\ \alpha} = ? \neq 0$ fotonun kütlesiz oluşundan dolayı.

Simetrik, izsiz, ayar değişmez gerilim tensörü $\Theta^{\alpha\beta}$

$$\begin{cases} T^{\alpha\beta} = \dfrac{-1}{4\pi} g^{\alpha\mu} F_{\mu\lambda} \partial^\beta A^\lambda - g^{\alpha\beta} \mathcal{L} \\ F^{\lambda\beta} = \partial^\lambda A^\beta - \partial^\beta A^\lambda \end{cases} \qquad \partial^\beta A^\lambda = \partial^\lambda A^\beta - F^{\lambda\beta}$$

$$\Rightarrow T^{\alpha\beta} = \frac{1}{4\pi}\left[g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} - \frac{1}{4\pi} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right] \qquad + \frac{1}{8\pi} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \quad \cancel{\mathcal{L}}$$

$$- \frac{1}{4\pi} g^{\alpha\mu} F_{\mu\lambda} \partial^\lambda A^\beta$$

1. terim.
[ ] α ve β ya göre sim. ve ayar değişmez.

$$T_D^{\alpha\beta} = \frac{-1}{4\pi} g^{\alpha\mu} F_{\mu\lambda} \partial^\lambda A^\beta$$

$$F_{\mu\lambda} = g_{\mu\sigma} g_{\lambda\gamma} F^{\sigma\gamma}$$

$$= \frac{-1}{4\pi} \underline{g^{\alpha\mu}}\, \underline{g_{\mu\sigma}}\, \underline{g_{\lambda\gamma}}\, \underline{F^{\sigma\gamma}}\, \underline{\partial^\lambda}\, \underline{A^\beta}$$

$$= \frac{-1}{4\pi} g_{\lambda\gamma} F^{\alpha\gamma} \partial^\lambda A^\beta = \frac{-1}{4\pi} F^{\alpha\gamma} \partial_\gamma A^\beta = \frac{-1}{4\pi} F^{\alpha\lambda} \partial_\lambda A^\beta$$

Maxwell denklemlerinden,

$$\partial^\beta F_{\beta\alpha} = \frac{4\pi}{c} J_\alpha = 0 \quad (\text{kaynak yok})$$

$$T_D^{\alpha\beta} = \frac{1}{4\pi}\left(F^{\lambda\alpha}\partial_\lambda A^\beta + A^\beta \underbrace{\partial_\lambda F^{\lambda\alpha}}_{0}\right)$$

$$= \frac{1}{4\pi}\partial_\lambda\left(A^\beta F^{\lambda\alpha}\right) \checkmark$$

i) $\partial_\alpha T_D^{\alpha\beta} = 0 = \frac{1}{4\pi}\partial_\alpha\partial_\lambda\left(A^\beta F^{\lambda\alpha}\right) = \frac{1}{8\pi}\left(\partial_\alpha\partial_\lambda A^\beta F^{\lambda\alpha} + \partial_\lambda\partial_\alpha A^\beta F^{\alpha\lambda}\right)$

ii) $\int T_D^{\alpha\beta} d^3x = 0 = \int T_D^{00} d^3x + \int T_D^{0i} d^3x \rightarrow \frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}$

$$\boxed{\Theta^{\alpha\beta} = T^{\alpha\beta} - T_D^{\alpha\beta}}$$

Θ^{00} enerji yoğunluğu

Θ^{0i} Poynting vek. nün i. bileşeni

Θ^{ij} Maxwell stres tensörü

$$\partial_\alpha \Theta^{\alpha\beta} = 0 \quad (\text{korunum yasası})$$

$\beta = 0 \Rightarrow \partial_\alpha \Theta^{\alpha 0} = 0 \Rightarrow \partial_0 \Theta^{00} + \partial_i \Theta^{i0} = 0$

$$\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = 0$$

$\beta = i \Rightarrow 0 = \partial_\alpha \Theta^{\alpha i} = \frac{\partial g_i}{\partial t} - \sum_{j>i}^{3} \partial_j T_{ij}^{(M)}$

(c.) Yüklü parçacıklar ile etkileşen elektromagnetik alanlar için korunum yasaları

$\Theta^{\alpha\beta}$ aynıdır. $\partial_\alpha \Theta^{\alpha\beta} \neq 0$ olur.

$$\partial_\alpha \Theta^{\alpha\beta} = \frac{1}{4\pi}\left[(\partial^\mu F_{\mu\lambda}) F^{\lambda\beta} + F_{\mu\lambda}(\partial^\mu F^{\lambda\beta}) + \underbrace{\frac{1}{2} F^{\mu\nu}\partial^\beta F_{\mu\nu}}_{\frac{1}{2}(\partial^\beta F^{\mu\nu}) F_{\mu\nu}} \right]$$

$$\partial^\mu F_{\mu\lambda} = \frac{4\pi}{c} J_\lambda$$

$$\partial_\alpha \Theta^{\alpha\beta} + \frac{1}{c} J_\lambda F^{\beta\lambda} = \frac{1}{8\pi} F_{\mu\lambda}\left[\partial^\mu F^{\lambda\beta} + \partial^\mu F^{\lambda\beta} + \partial^\beta F^{\mu\lambda} \right]$$

$$\partial^\mu F^{\lambda\beta} + \partial^\beta F^{\mu\lambda} + \partial^\lambda F^{\beta\mu} = 0 \qquad \text{HMD}$$

$$\text{LHS} = \frac{1}{8\pi} F_{\mu\lambda}\left[\partial^\mu F^{\lambda\beta} - \partial^\lambda F^{\beta\mu} \right]$$

$$\partial_\alpha \Theta^{\alpha\beta} + \frac{1}{c} J_\lambda F^{\beta\lambda} = \frac{1}{8\pi} F_{\mu\lambda}\left(\partial^\mu F^{\lambda\beta} + \partial^\lambda F^{\mu\beta} \right) = 0$$

$$\boxed{\partial_\alpha \Theta^{\alpha\beta} = \frac{1}{c} J_\lambda F^{\lambda\beta}}$$

$\underline{\underline{\beta = 0}}$

$$\partial_0 \Theta^{00} + \partial_i \Theta^{i0} = \frac{1}{c} J_0 F^{00} + \frac{1}{c} J_i F^{i0}$$

$$\Downarrow$$

$$\frac{1}{c}\frac{\partial u}{\partial t} + \frac{1}{c}\vec{\nabla}\cdot\vec{S} = -\frac{1}{c}\vec{J}\cdot\vec{E}$$

$$\boxed{\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}}$$ Poynting teoremi

$$\partial_\alpha \Theta^{\alpha\beta} = \underbrace{\frac{-1}{c} J_\lambda F^{\beta\lambda}}_{}$$

$$f^\beta = \left(\frac{1}{c} \vec{J} \cdot \vec{E} \,,\, \rho \vec{E} + \frac{1}{c} \vec{J} \times \vec{B} \right)$$

$\beta = 0 \qquad \beta = i$

$$\int f^\beta d^3x = \frac{dP^\beta_{\text{particle}}}{dt}$$

$$\int \partial_\alpha \Theta^{\alpha\beta} d^3x = \frac{dP^\beta_{\text{field}}}{dt}$$

$$\frac{d}{dt}\left(P^\beta_{\text{particle}} + P^\beta_{\text{field}} \right) = 0$$

BS K

$L \to -L$ yeni baskıda!

12.1. (a) $L = \frac{1}{2} m U^\alpha U_\alpha + \frac{q}{c} U_\alpha A^\alpha$ $\left\{ \begin{array}{l} U^\alpha = (\gamma c, \gamma \vec{v}) \\ A^\alpha = (\Phi, \vec{A}) \end{array} \right.$

Lorentz invaryant Lagranjiyenin A^α tarafından tarif edilen bir dış alan ile etkileşen q yüklü ve m kütleli bir parçacık için doğru relativistik hareket denklemlerini verdiğini gösteriniz. (b) kanonik mom.'u tanımlayınız ve etkin Hamiltoniyeni hem kovaryant ve hem de uzay-zaman formunda yazınız.

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial U_\beta}\right) - \frac{\partial L}{\partial x_\beta} = 0$$

$$\frac{\partial L}{\partial U_\beta} = m U^\beta + \frac{q}{c} A^\beta = P^\beta$$ kanonik momentum

$$\frac{\partial L}{\partial x_\beta} = \frac{q}{c} U_\alpha \frac{\partial A^\alpha}{\partial x_\beta} = \frac{q}{c} U_\alpha \partial^\beta A^\alpha$$

$$\Rightarrow \frac{d}{d\tau}\left(m U^\beta + \frac{q}{c} A^\beta\right) = \frac{q}{c} U_\alpha \partial^\beta A^\alpha$$

$$m \frac{dU^\beta}{d\tau} + \frac{q}{c} \frac{dx_\alpha}{d\tau} \frac{\partial A^\beta}{\partial x_\alpha} = \frac{q}{c} U_\alpha \partial^\beta A^\alpha$$

$$m \frac{dU^\beta}{d\tau} = \frac{q}{c} U_\alpha \left(\partial^\beta A^\alpha - \partial^\alpha A^\beta\right)$$

$$= \frac{q}{c} U_\alpha F^{\alpha\beta}$$

(b) $P^\alpha = m U^\alpha + \frac{q}{c} A^\alpha$ (kanonik mom.'u belirleyiniz ve ef. Ham. yeni hem kovaryant ve hem de uzay-zaman formunda yazınız.

$$H = U_\beta P^\beta - L = U_\beta \left(m U^\beta + \frac{q}{c} A^\beta\right) - \frac{1}{2} m U_\beta U^\beta - \frac{q}{c} U_\beta A^\beta$$

$$= \frac{1}{2} m U_\beta U^\beta = -\frac{1}{2} m c^2$$

$$\frac{ceE_0t}{\mathcal{E}_0} = \sinh\frac{y}{c^2p_0} \quad \text{ise} \quad e^2E_0^2x^2 = \mathcal{E}_0^2 + (ecE_0t)^2$$

$$= \mathcal{E}_0^2\left(1+\sinh^2\frac{y}{c^2p_0}\right)$$

$$= \mathcal{E}_0^2\cosh^2\frac{y}{c^2p_0}$$

$$\boxed{x = \frac{\mathcal{E}_0}{eE_0}\cosh\frac{y}{c^2p_0}}$$

$$x = \sqrt{\left(\frac{\mathcal{E}_0}{eE_0}\right)^2 + (ct)^2} \qquad ct \ll \frac{\mathcal{E}_0}{eE_0} \text{ ise}$$

$$x = \frac{\mathcal{E}_0}{eE_0}\left[1+\left(\frac{ct}{\mathcal{E}/eE_0}\right)^2\right]^{1/2} = \frac{\mathcal{E}_0}{eE_0}\left[1+\frac{1}{2}\left(\frac{eE_0ct}{\mathcal{E}}\right)^2 \cancel{+\dots}\right]$$

$$\begin{cases} x = \dfrac{\mathcal{E}_0}{eE_0} \\ y = \dfrac{p_0}{m}t \end{cases} \qquad x\text{-}y\text{'de yörünge parabol.}$$

$$ct \gg \frac{\mathcal{E}_0}{eE_0} \text{ ise} \qquad x = ct\sqrt{1+\left(\frac{\mathcal{E}_0/eE_0}{ct}\right)^2} = ct\left[1+\frac{1}{2}\left(\frac{\mathcal{E}_0/eE_0}{ct}\right)^2+\dots\right]$$

$$= ct$$

$$y \cong \frac{p_0c}{eE_0}\ln\frac{ceE_0t}{\mathcal{E}_0}$$

$$(D^2 - i\lambda B' D)\, r = 0$$

$$D(D - i\lambda B')\, r = 0 \quad \exists \quad r = C' \frac{e^{i\lambda B'\tau}}{i\lambda B'} + C_2$$

$$\begin{cases} x' = \dfrac{C_2}{\lambda B'} \sin \lambda B'\tau + C_2 \\[2ex] z' = \dfrac{C_2}{\lambda B'} \cos \lambda B'\tau \end{cases}$$

Buradan K'ya geçebiliriz

$$x = \frac{C_2}{\lambda B'} \sin \lambda B'\tau + C_2'$$

$$y = C_1'\tau$$

$$z = \frac{B}{\sqrt{B^2 - E^2}} \left\{ \frac{C_2}{\lambda B'} \cos \lambda B'\tau + C_2' + \frac{E}{B} C_1 \tau \right\}$$

$$ct = \frac{B}{\sqrt{B^2 - E^2}} \left\{ C_1 \tau + \frac{E}{B} \frac{C_2}{\lambda B'} \sin \lambda B'\tau + C_2' \right\}$$

$$U^\alpha U_\alpha = c^2 \qquad C_1^2 - C_2^2 - C_1'^2 = c^2$$

$$ct = \gamma\,(ct' + \beta z')$$

bunun tersi de yapılabilir

b) $\theta = 0$ ise,

$$\begin{cases} c\ddot{t} = -\lambda E \dot{z} \\ \ddot{x} = \lambda B \dot{y} \\ \ddot{y} = -\lambda B \dot{x} \\ \ddot{z} = -\lambda E c \dot{t} \end{cases}$$

$$\ddot{x} + i\ddot{y} = -i\lambda B (\dot{x} + i\dot{y}) \Rightarrow \ddot{r} = -i\lambda B \dot{r}$$

$$\dot{r} = a e^{-i\lambda B\tau} \quad \begin{matrix} \nearrow \dot{x} = a \cos\lambda B\tau \\ \searrow \dot{y} = -a \sin\lambda B\tau \end{matrix} \quad \begin{cases} x = \dfrac{a}{\lambda B} \sin\lambda B\tau \\ y = \dfrac{a}{\lambda B} \cos\lambda B\tau \end{cases}$$

$$\ddot{z} = -\lambda E c\ddot{t} = -\lambda E(-\lambda E)\dot{z}$$

$$\dddot{z} - (\lambda E)^2 \dot{z} = 0 \longrightarrow (D^3 - (\lambda E)^2 D) z = 0$$

$$D(D - \lambda E)(D + \lambda E) z = 0$$

$$z = \cancel{c_1} + c_2 e^{\lambda E\tau} + c_3 e^{-\lambda E\tau}$$

$$= b \cosh \lambda E\tau \qquad \tau = 0 \text{ da } z = 0 \text{ kabul ettik.}$$

$$c\dot{t} = -b \sinh \lambda E\tau$$

$$U^\alpha U_\alpha = c^2 \Rightarrow (\lambda E)^2 b^2 \cosh^2 \lambda\tau E - a^2 \cos^2 \lambda B\tau - a^2 \sin^2 \lambda B\tau$$

$$= -b^2 (\lambda E)^2 \sinh^2 (\lambda E\tau) = 1$$

$$(\lambda E b)^2 - a^2 = 1$$

$$b = \frac{\sqrt{a^2+1}}{\lambda E} \qquad R = \frac{mc^2}{eB} \quad \rho = \frac{E}{B} \quad \phi = \frac{c}{R}\tau = \lambda B\tau$$

d) $L^\alpha = (\gamma\beta, \gamma\hat{\beta})$

$N^\alpha = (0, \hat{n})$

$$\frac{d\theta}{d\tau} = \frac{e\gamma}{mc}\left[\left(\frac{g}{2}-1\right)\hat{n}\cdot(\hat{\beta}\times\vec{B}) + \left(\frac{g\beta}{2}-\frac{1}{\beta}\right)\hat{n}\cdot\vec{E}\right]$$

$$= \frac{e}{mc}\left[\frac{g}{2}\gamma\beta\,\hat{n}\cdot\vec{E} + \frac{g}{2}\gamma\hat{n}\cdot(\hat{\beta}\times\vec{B}) - \frac{\gamma}{\beta}\hat{n}\cdot\vec{E} - \gamma\hat{n}\cdot(\vec{\beta}\times\vec{B})\right]$$

$$= \frac{e}{mc}\left[\frac{g}{2}L_\alpha - \frac{1}{v}U_\alpha\right]F^{\alpha\beta}N_\beta$$

12.16) 4-boyutta diverjans teoremi ile kaynaksız Θ^{00} ve Θ^{0i}'nin 3-boyutlu uzay integralleri

$$\int \Theta^{00} d^3x \quad , \quad \int \Theta^{0i} d^3x$$

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\left[g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}\right]$$

$$\partial_\alpha \Theta^{\alpha\beta} = 0$$

$$\Theta^{00} = \frac{1}{4\pi}\left[F_{0\lambda}F^{\lambda 0} + \frac{1}{4}F_{\mu\lambda}F^{\mu\lambda}\right] = \frac{1}{8\pi}(E^2+B^2)$$

$$\frac{1}{8\pi}\int (E^2+B^2)\, d^3x = E$$

$$\Theta^{0i} = \frac{1}{4\pi}(\vec{E}\times\vec{B})_i$$

$$\frac{1}{4\pi}\int (\vec{E}\times\vec{B})_i\, d^3x = c^2 P_i$$