

XII ZAMAN GELİŞİMİ (EVİRİMİ)

Zamanı bir dizi durumu etiketleyen sürekli gerçel bir parametre olarak varsayacağız.

$W(t_0)$: $t=t_0$ zamanında hazırlanmış bir sist.'in durumunu gösterir.

- Daha sonraki bir zamanda $t=t_1 > t_0$ $W(t_0)$, $W(t_1)$ 'i tek olarak belirler. Evrim esnasında, sistem m. etkilere maruz kalabilir, öyle ki evrim t_0 'dan t_1 'e olan aralığa ağırlıkla bağlıdır. İlk olarak m. etkilere bakatın etkinliği? (bu sistem) olarak evrim gelişen sist.'i ele alacağız. Bu gibi sist.'ler için

$$W(t_0) \rightarrow W(t_1) \quad \text{evrim,} \quad 1.1$$

t_0 'a bağlı, ancak sist.'in $W(t_0)$ başlangıç durumu ve $Z_1 = t_1 - t_0$ 'a bağlıdır.

1. Gereklikler:

- ① (1.1.) ifadesi durumlar kümesinin bir lineer dönüşümdür.

$$W_1(t_0) \rightarrow W_1(t_1) \quad \text{ve} \quad W_2(t_0) \rightarrow W_2(t_1) \quad \text{ile}$$

$$a, b \in \mathbb{R} \quad \text{için} \quad aW_1(t_0) + bW_2(t_0) \rightarrow aW_1(t_1) + bW_2(t_1)$$

- ② Zaman evrimi, toplama göre bir grup oluşturur.

$$W(t_0) \rightarrow W(t_0 + \tau_1) \quad \tau_1 = t_1 - t_0 \quad \text{için}$$

$$W(t_1) \rightarrow W(t_1 + \tau_2) \quad \tau_2 = t_2 - t_1 \quad \text{için de zaman}$$

$$W(t_0) \rightarrow W(t_0 + \tau_1 + \tau_2) \text{ olur.}$$

1. ve 2. gereksinimler tersinir süreçler tarafından da sağlanabilir. Burada tersinir süreçlerin R.M. ile ilgileniyorum ve bu gibi süreçler için ek bir gereksinim:

③ (1.1) zaman evrimi $U(z)$ üniter op. lü ile verilir: ve bu op. şunları sağlar

$$W(t_0) \rightarrow W(t_1) = U^\dagger(z_1) W(t_0) U(z_1) \quad 1.2$$

$$U^\dagger(z_1) = U^{-1}(z_1) \quad 1.3$$

$$U(0) = I \quad 1.4$$

$$U^\dagger(z) = U(-z) \quad 1.5$$

$$U(z_1 + z_2) = U(z_1) U(z_2) \quad -\infty < z_1, z_2 < \infty \quad 1.6$$

$$U(z) \text{ } z \text{ parametresinin süreci bir op. fkt. 'udur.} \quad 1.7$$

⑧ (1.5.) $W(t_1) = U^\dagger(z_1) W(t_0) U(z_1)$

$$\Rightarrow U(z_1) W(t_1) U^{-1}(z_1) = W(t_0)$$

zamanla geriye doğru evrim

$$W(t_0) = U^{-1}(-z_1) W(t_1) U(-z_1)$$

$$U^{-1}(z) = U(-z)$$

⑧ (1.1.6.) gereksinim (2) $W(t_0) \rightarrow W(t_0 + \tau_1 + \tau_2)$

$$U(t_2 - t_0) = U(t_1 - t_0) U(t_2 - t_1) \quad U(z_1 + z_2) = U(z_1) U(z_2)$$

$$z_1 = t_1 - t_0 \quad z_2 = t_2 - t_1$$

BSK 1.1.6.1.2.1.3.4.5.6.7.8.9.10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.25.26.27.28.29.30.31.32.33.34.35.36.37.38.39.40.41.42.43.44.45.46.47.48.49.50.51.52.53.54.55.56.57.58.59.60.61.62.63.64.65.66.67.68.69.70.71.72.73.74.75.76.77.78.79.80.81.82.83.84.85.86.87.88.89.90.91.92.93.94.95.96.97.98.99.100.101.102.103.104.105.106.107.108.109.110.111.112.113.114.115.116.117.118.119.120.121.122.123.124.125.126.127.128.129.130.131.132.133.134.135.136.137.138.139.140.141.142.143.144.145.146.147.148.149.150.151.152.153.154.155.156.157.158.159.160.161.162.163.164.165.166.167.168.169.170.171.172.173.174.175.176.177.178.179.180.181.182.183.184.185.186.187.188.189.190.191.192.193.194.195.196.197.198.199.200.201.202.203.204.205.206.207.208.209.210.211.212.213.214.215.216.217.218.219.220.221.222.223.224.225.226.227.228.229.230.231.232.233.234.235.236.237.238.239.240.241.242.243.244.245.246.247.248.249.250.251.252.253.254.255.256.257.258.259.260.261.262.263.264.265.266.267.268.269.270.271.272.273.274.275.276.277.278.279.280.281.282.283.284.285.286.287.288.289.290.291.292.293.294.295.296.297.298.299.300.301.302.303.304.305.306.307.308.309.310.311.312.313.314.315.316.317.318.319.320.321.322.323.324.325.326.327.328.329.330.331.332.333.334.335.336.337.338.339.340.341.342.343.344.345.346.347.348.349.350.351.352.353.354.355.356.357.358.359.360.361.362.363.364.365.366.367.368.369.370.371.372.373.374.375.376.377.378.379.380.381.382.383.384.385.386.387.388.389.390.391.392.393.394.395.396.397.398.399.400.401.402.403.404.405.406.407.408.409.410.411.412.413.414.415.416.417.418.419.420.421.422.423.424.425.426.427.428.429.430.431.432.433.434.435.436.437.438.439.440.441.442.443.444.445.446.447.448.449.450.451.452.453.454.455.456.457.458.459.460.461.462.463.464.465.466.467.468.469.470.471.472.473.474.475.476.477.478.479.480.481.482.483.484.485.486.487.488.489.490.491.492.493.494.495.496.497.498.499.500.501.502.503.504.505.506.507.508.509.510.511.512.513.514.515.516.517.518.519.520.521.522.523.524.525.526.527.528.529.530.531.532.533.534.535.536.537.538.539.540.541.542.543.544.545.546.547.548.549.550.551.552.553.554.555.556.557.558.559.560.561.562.563.564.565.566.567.568.569.570.571.572.573.574.575.576.577.578.579.580.581.582.583.584.585.586.587.588.589.590.591.592.593.594.595.596.597.598.599.600.601.602.603.604.605.606.607.608.609.610.611.612.613.614.615.616.617.618.619.620.621.622.623.624.625.626.627.628.629.630.631.632.633.634.635.636.637.638.639.640.641.642.643.644.645.646.647.648.649.650.651.652.653.654.655.656.657.658.659.660.661.662.663.664.665.666.667.668.669.670.671.672.673.674.675.676.677.678.679.680.681.682.683.684.685.686.687.688.689.690.691.692.693.694.695.696.697.698.699.700.701.702.703.704.705.706.707.708.709.710.711.712.713.714.715.716.717.718.719.720.721.722.723.724.725.726.727.728.729.730.731.732.733.734.735.736.737.738.739.740.741.742.743.744.745.746.747.748.749.750.751.752.753.754.755.756.757.758.759.760.761.762.763.764.765.766.767.768.769.770.771.772.773.774.775.776.777.778.779.780.781.782.783.784.785.786.787.788.789.790.791.792.793.794.795.796.797.798.799.800.801.802.803.804.805.806.807.808.809.810.811.812.813.814.815.816.817.818.819.820.821.822.823.824.825.826.827.828.829.830.831.832.833.834.835.836.837.838.839.840.841.842.843.844.845.846.847.848.849.850.851.852.853.854.855.856.857.858.859.860.861.862.863.864.865.866.867.868.869.870.871.872.873.874.875.876.877.878.879.880.881.882.883.884.885.886.887.888.889.890.891.892.893.894.895.896.897.898.899.900.901.902.903.904.905.906.907.908.909.910.911.912.913.914.915.916.917.918.919.920.921.922.923.924.925.926.927.928.929.930.931.932.933.934.935.936.937.938.939.940.941.942.943.944.945.946.947.948.949.950.951.952.953.954.955.956.957.958.959.960.961.962.963.964.965.966.967.968.969.970.971.972.973.974.975.976.977.978.979.980.981.982.983.984.985.986.987.988.989.990.991.992.993.994.995.996.997.998.999.1000.1001.1002.1003.1004.1005.1006.1007.1008.1009.1010.1011.1012.1013.1014.1015.1016.1017.1018.1019.1020.1021.1022.1023.1024.1025.1026.1027.1028.1029.1030.1031.1032.1033.1034.1035.1036.1037.1038.1039.1040.1041.1042.1043.1044.1045.1046.1047.1048.1049.1050.1051.1052.1053.1054.1055.1056.1057.1058.1059.1060.1061.1062.1063.1064.1065.1066.1067.1068.1069.1070.1071.1072.1073.1074.1075.1076.1077.1078.1079.1080.1081.1082.1083.1084.1085.1086.1087.1088.1089.1090.1091.1092.1093.1094.1095.1096.1097.1098.1099.1100.1101.1102.1103.1104.1105.1106.1107.1108.1109.1110.1111.1112.1113.1114.1115.1116.1117.1118.1119.1120.1121.1122.1123.1124.1125.1126.1127.1128.1129.1130.1131.1132.1133.1134.1135.1136.1137.1138.1139.1140.1141.1142.1143.1144.1145.1146.1147.1148.1149.1150.1151.1152.1153.1154.1155.1156.1157.1158.1159.1160.1161.1162.1163.1164.1165.1166.1167.1168.1169.1170.1171.1172.1173.1174.1175.1176.1177.1178.1179.1180.1181.1182.1183.1184.1185.1186.1187.1188.1189.1190.1191.1192.1193.1194.1195.1196.1197.1198.1199.1200.1201.1202.1203.1204.1205.1206.1207.1208.1209.1210.1211.1212.1213.1214.1215.1216.1217.1218.1219.1220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.2221.2222.2223.2224.2225.2226.2227.2228.2229.2230.2231.2232.2233.2234.2235.2236.2237.2238.2239.2240.2241.2242.2243.2244.2245.2246.2247.2248.2249.2250.2251.2252.2253.2254.2255.2256.2257.2258.2259.2260.2261.2262.2263.2264.2265.2266.2267.2268.2269.2270.2271.2272.2273.2274.2275.2276.2277.2278.2279.2280.2281.2282.2283.2284.2285.2286.2287.2288.2289.2290.2291.2292.2293.2294.2295.2296.2297.2298.2299.2300.2301.2302.2303.2304.2305.2306.2307.2308.2309.2310.2311.2312.2313.2314.2315.2316.2317.2318.2319.2320.2321.2322.2323.2324.2325.2326.2327.2328.2329.2330.2331.2332.2333.2334.2335.2336.2337.2338.2339.2340.2341.2342.2343.2344.2345.2346.2347.2348.2349.2350.2351.2352.2353.2354.2355.2356.2357.2358.2359.2360.2361.2362.2363.2364.2365.2366.2367.2368.2369.2370.2371.2372.2373.2374.2375.2376.2377.2378.2379.2380.2381.2382.2383.2384.2385.2386.2387.2388.2389.2390.2391.2392.2393.2394.2395.2396.2397.2398.2399.2400.2401.2402.2403.2404.2405.2406.2407.2408.2409.2410.2411.2412.2413.2414.2415.2416.2417.2418.2419.2420.2421.2422.2423.2424.2425.2426.2427.2428.2429.2430.2431.2432.2433.2434.2435.2436.2437.2438.2439.2440.2441.2442.244

Tanım: $A \equiv \left. \frac{dU(z)}{dz} \right|_{z=0}$

sonsuz küçük op. ya da $U(z)$ zaman aralığı op.'nin generatörü

$U(z)$ sadece türetilenler aynı zamanda herfi sayı da türetilenlerdir. 1.6 ya τ_1 'e göre türet.

$$\frac{dU(\tau+\tau_1)}{d(\tau+\tau_1)} \frac{d(\tau+\tau_1)}{d\tau_1} = \frac{dU(\tau_1)}{d\tau_1} U(\tau)$$

$\tau_1=0 \Rightarrow \frac{dU(z)}{dz} = A U(z)$

$\frac{d^2 U(z)}{dz^2} = A^2 U(z) \quad \dots \quad \frac{d^p U(z)}{dz^p} = A^p U(z)$

$U(0) = I$ için $\left. \frac{d^p U(z)}{dz^p} \right|_{z=0} = A^p$

$U(z)$ 'nin Taylor açılımı:

$$U(z) = I + \tau A + \frac{\tau^2}{2!} A^2 + \dots + \frac{\tau^n}{n!} A^n + \dots$$

$U(z) = e^{\tau A}$

$U(z) |a\rangle = e^{\tau a} |a\rangle \quad \{ A |a\rangle = a |a\rangle \text{ için} \}$

$U^\dagger(z) = e^{\tau A^\dagger}$

$$U^\dagger(z) = I + \tau A^\dagger + \frac{\tau^2}{2!} A^{\dagger 2} + \dots + \frac{\tau^n}{n!} A^{\dagger n} + \dots$$

$$U^\dagger(z) U(z) = I$$

$$\frac{dU^\dagger(z)}{dz} U(z) + U^\dagger(z) \frac{dU(z)}{dz} = 0 \quad \left| \right|_{z=0} \quad \text{el.}$$

$$A^\dagger U(0) = -U^\dagger(0) A$$

$$A^\dagger U(0) = -U^{-1}(0) A \Rightarrow A^\dagger = -A \quad \text{i.i.}$$

○ Show - Hermitian.

$$H = -i\hbar A = -i\hbar \left. \frac{dU(z)}{dz} \right|_{z=0} \quad \text{Zaman evrimi op. lü.}$$

$$A = -\frac{1}{i\hbar} H = \frac{i}{\hbar} H$$

~~ka~~ Schrödinger Temini : (Resmi)

Korunumlu bir fiziksel sist. , gözlenebilirler cinsinde Hermitian bir eleman olan ve fiziksel ist. 'in karakteristiği olan zaman öteleme jeneratörü H'ye sahiptir. Fiziksel durumu zaman evrimi

$$W(t) = U^\dagger(t) W_0 U(t)$$

ile verilir. burada

W_0 : $t=0$ başlangıçta sist. 'in durumu

$$U(t) = e^{iHt/\hbar}$$

A 'nin beklenen değeri,

$$\langle A \rangle = \text{Tr}(AW)$$

$$\langle A \rangle_{t=t_0} \neq \langle A \rangle_{t=t_1}$$

$$\langle A \rangle_t = \text{Tr}(AW(t)) = \text{Tr}(AU^\dagger(t)W_0U(t))$$

$$= \text{Tr} \left(U(t)AU^\dagger(t)W_0 \right) = \text{Tr}(A(t)W_0)$$

$$\bigcirc \quad A(t) = U(t)AU^\dagger(t)$$

~~Vb~~ Heisenberg Temeli: (Remi)

Komünikasyon fiziksel bir sist., gözlenesilirler cebirinin Hermitel bir elemanı olan ve fiziksel sist. ile karakteristiği olan zaman öteleme için bir π denatörüne sahiptir. Fiziksel sistem her $A(t)$ gözlenesilirlik zaman evrimi gelişimi

$$A(t) = U(t)AU^\dagger(t)$$

1.22

ile verilir; burada

A , $t_0=0$ da gözlenesilirlik

$$U(t) = e^{i\hbar H/t}$$

1.22. den bir gözlenesilirlik zaman seyahatini elde ederiz:

$$\frac{dA(t)}{dt} = \frac{i}{\hbar} \hbar e^{i\hbar H/t} A e^{-i\hbar H/t} + e^{i\hbar H/t} A \left(-\frac{i\hbar}{\hbar} \right) e^{-i\hbar H/t}$$

$$\text{BS K} \quad = \frac{i}{\hbar} (\hbar A(t) - A(t)\hbar) =$$

$$i\hbar \frac{dA(t)}{dt} = [A(t), H]$$

1.24.

Heisenberg hareket Denk.'i. Birim 5 optayonlar hareket d'd'd.
H'i'i i'eren herhangi CSCO 'un eleman b' hareket ist.'alr.

Fiziksel bir sist. i'inde P_i ve Q_i g'oklenesizlikleri alalm. Her
b'isi ne hareket ist. deg'ildir.

$$t=0 \text{ da } [P_i, Q_j] = \frac{\hbar}{i} \delta_{ij} I$$

$$t>0 \text{ da } P_i(t), Q_j(t) - Q_j(t), P_i(t)$$

$$= u(t) P_i u^\dagger(t) u(t) Q_j u^\dagger(t) - u(t) Q_j u^\dagger(t) u(t) P_i u^\dagger(t)$$

$$= u(t) (P_i Q_j - Q_j P_i) u^\dagger(t) = u(t) u^\dagger(t) \frac{\hbar}{i} \delta_{ij} I$$

$$= \frac{\hbar}{i} \delta_{ij} I$$

$$[P_i(t), Q_j(t)] = \frac{\hbar}{i} \delta_{ij} I$$

hareket denklemleri,

$$H = \frac{1}{2m} P_i P_i + V(\vec{Q}) \quad Q_i = Q_i(t) \quad P_i = P_i(t)$$

$$[Q_k, H] = \frac{1}{2m} [Q_k, P_i P_i] = \frac{1}{2m} \{ [Q_k, P_i], P_i \}$$

$$= \frac{i\hbar}{2m} \delta_{ki} \{ I, P_i \} = \frac{i\hbar}{m} P_k$$

$$\Rightarrow \frac{dQ_k(t)}{dt} = \frac{P_k(t)}{m} \quad \text{Her bir 5 optayonun Q. H. hareketi}$$

klassik fürchte $f_k = - \frac{\partial V(\vec{x})}{\partial x_k}$

Q.N. $F_k \equiv -\frac{i}{\hbar} [P_k, V(\vec{Q})] = - \frac{\partial V(\vec{Q})}{\partial Q_k}$ (1.27.)

) komut op.

$$\frac{d P_k(t)}{dt} = \frac{1}{i\hbar} [P_k, H] = -\frac{i}{\hbar} [P_k, V(\vec{Q})]$$

$$= F_k(\vec{Q}(t))$$

in $\frac{d^2 Q_k(t)}{dt^2} = F_k(\vec{Q}(t))$ Q.N. Newton dardini.

[" $\frac{\partial V(\vec{Q})}{\partial Q_k}$ "] : $V(\vec{Q})$ de Q_i 'leni x_i ile deqistil
 $V(\vec{x})$: x_k ye qare tinst. $\partial/\partial x_k$ de
 x_i 'ni Q_i ile deqistil.

1.27. 'nt cene conlizi :

1. $V(\vec{Q}) = C + C_1 Q_1 + C_2 Q_1 Q_2 + \dots + C_{1\dots i_n} Q_{i_1} \dots Q_{i_n}$

$C_{i_1\dots i_n}$ 'en simetrik : $C_{i_1\dots i_p\dots i_q\dots i_n} = C_{i_1\dots i_q\dots i_p\dots i_n}$

$$\frac{i}{\hbar} [P_k, V(\vec{Q})] = C_k + 2 C_{ki} Q_i + \dots + n C_{ki_2 i_3 \dots i_n} Q_{i_2} \dots Q_{i_n} + \dots$$

çünkü

$$C_{i_1 \dots i_n} [P_L, Q_{i_1 \dots i_n}]$$

$$= C_{i_1 \dots i_n} \sum_{p=1}^n [P_L, Q_{i_p}] Q_{i_1} \dots Q_{i_{p-1}} Q_{i_{p+1}} \dots Q_{i_n}$$

$$= C_{i_1 \dots i_n} \sum_{p=1}^n \frac{\hbar}{i} \delta_{k i_p} Q_{i_1} \dots Q_{i_{p-1}} Q_{i_{p+1}} \dots Q_{i_n}$$

$$0 = n \frac{\hbar}{i} C_{k i_2 \dots i_n} Q_{i_2} Q_{i_3} \dots Q_{i_n}$$

2. $[P_L, V(\vec{Q})]$ 'yı hesaplamamıza bakalım

$$\langle \vec{x} | [P_L, V(\vec{Q})] | \vec{x} \rangle = \langle \vec{x} | \underbrace{P_L}_{\text{operatör}} (V(\vec{Q})) | \vec{x} \rangle$$

$$- \langle \vec{x} | \underbrace{V(\vec{Q})}_{\text{operatör}} \underbrace{P_L}_{\text{operatör}} | \vec{x} \rangle$$

$$= \frac{\hbar}{i} \frac{\partial}{\partial x_k} \langle \vec{x} | V(\vec{Q}) | \vec{x} \rangle - V(\vec{x}) \frac{\hbar}{i} \frac{\partial}{\partial x_k} \langle \vec{x} | \vec{x} \rangle$$

$$= \frac{\hbar}{i} \frac{\partial}{\partial x_k} (V(\vec{x}) \langle \vec{x} | \vec{x} \rangle) - V(\vec{x}) \frac{\hbar}{i} \frac{\partial}{\partial x_k} \langle \vec{x} | \vec{x} \rangle$$

$$= \frac{\hbar}{i} \frac{\partial V(\vec{x})}{\partial x_k} \langle \vec{x} | \vec{x} \rangle$$

↓

~~Me~~. Kommutator physikalisch ist. in Zusammenhang, um
gözeigenschaften desinb sichele symetri dörüüü
de verble.

Schwingen tennti

$W(t) = U^\dagger(t) W_0 U(t)$ ist nun auf. form'e geordn.

$$\bigcirc \quad \frac{dW}{dt} = \frac{i}{\hbar} (W(t) H - H W(t)) = \frac{i}{\hbar} [W(t), H]$$

W_0 soll dann sein:

$$W_0 = \Lambda_{|\psi_0\rangle} = |\psi_0\rangle\langle\psi_0|$$

$$W(t) = U^\dagger(t) \Lambda_{|\psi_0\rangle} U(t) = U^\dagger(t) |\psi_0\rangle\langle\psi_0| U(t)$$

$$\rightarrow |\psi(t)\rangle = U^\dagger(t) |\psi_0\rangle = e^{-iHt/\hbar} |\psi_0\rangle$$

$$\bigcirc \Rightarrow W(t) = U^\dagger(t) \Lambda_{|\psi_0\rangle} U(t) = |\psi(t)\rangle\langle\psi(t)| = \Lambda_{|\psi(t)\rangle}$$

$$\frac{d}{dt} |\psi(t)\rangle = \frac{dU^\dagger(t)}{dt} |\psi_0\rangle = -\frac{i}{\hbar} H U^\dagger(t) |\psi_0\rangle = -\frac{iH}{\hbar} |\psi(t)\rangle$$

$|\psi(t)\rangle$ nun auch: i. d. Schwingen dorkem

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

$$H = \frac{1}{2m} P_i P_i + V(\vec{Q})$$

$$i\hbar \frac{d}{dt} \langle \vec{x} | \psi(t) \rangle = \frac{1}{2m} \langle \vec{x} | P_i P_i | \psi(t) \rangle + V(\vec{x}) \langle \vec{x} | \psi(t) \rangle$$

$$= \frac{\hbar}{i} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} \langle \vec{x} | \psi(t) \rangle$$

$$\Rightarrow i\hbar \frac{d}{dt} \psi(\vec{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{x}, t) + V(\vec{x}) \psi(\vec{x}, t)$$

$$\psi(\vec{x}, t) = \langle \vec{x} | \psi(t) \rangle$$

Ex 1 : $\begin{array}{ll} \text{S.P} & Q + sbt \quad |\psi(t)\rangle \\ \text{H.P} & Q(t) \quad | \psi \rangle + sbt \end{array} \quad \left. \begin{array}{l} \text{Tr}(AW) \text{ 'un zman} \\ \text{scăpă lăși așchi} \end{array} \right\}$

Ex 2 de $A(t) |a\rangle = a |a\rangle$

$$U(t) |a_0\rangle = |a(t)\rangle = e^{i\hbar H/t} |a_0\rangle \quad \text{unde } |a_0\rangle \text{ sst tiber.}$$

Ex 3 'aa $|a_0\rangle$ sst tiber, $|\psi_0\rangle$ cum vechi și ter
având aceeași adre,

$$U^\dagger(t) |\psi_0\rangle = |\psi(t)\rangle = e^{-i\hbar H/t} |\psi_0\rangle$$

Acești doi operatori de transformare sunt egali și
dăruie să se verifice în celelalte zman scăpă lăși așchi d.

$$|\langle a(t) | \psi_0 \rangle|^2 = |\langle U(t) | a_0 \rangle, |\psi_0 \rangle|^2$$

$$BS \quad K = |\langle a_0 | U^\dagger(t) | \psi_0 \rangle|^2 = |\langle a_0 | \psi(t) \rangle|^2$$

Korollar: Zamanla değişmez ve bu anlamda korollarıdır.

$$W(t) = U^\dagger(t) W_0 U(t) = W_0 = W$$

$$\begin{aligned} \langle A \rangle_t &= \text{Tr}(A W(t)) = \text{Tr}(A U^\dagger(t) W U(t)) = \text{Tr}(A W) \\ &= \langle A \rangle_{t=0} \end{aligned}$$

- Korollar durumlar daima H enerji op.'nin öz durumlarıdır.

(∴) Günlü ; $W e^{iHt/\hbar} = e^{iHt/\hbar} W$

$$\rightarrow W H - H W = 0$$

Scf korollar durumu için

$$W = \Lambda_{|\psi_0\rangle} = |\psi_0\rangle \langle \psi_0|$$

○ $|\psi_0\rangle \langle \psi_0| H = H |\psi_0\rangle \langle \psi_0|$

$$H |\psi_0\rangle = |\psi_0\rangle \frac{\langle \psi_0 | H | \psi_0 \rangle}{\langle \psi_0 | \psi_0 \rangle}$$

(Scf durumlar enerji op.'nin
özvektörleri ve temel
eğerlerdir :

zaman scf. durumu:

$$|\psi(t)\rangle = U^\dagger(t) |\psi_0\rangle = e^{-iEt/\hbar} |\psi_0\rangle$$

~~Dura~~ Temel:

$$H = H_0 + H_1$$

$\underbrace{\hspace{1cm}}_{\text{tam}} \quad \underbrace{\hspace{1cm}}_{\text{etkilene}}$

H a.c ile zaman bağımlılığı yok ise !... (kuvvetli sist.)

$$U(t) = e^{iHt/\hbar}$$

0 zaman

$$\langle A \rangle_t = \text{Tr} (U(t) A U^\dagger(t) W_0)$$

Tanım:

$$\begin{cases} U_0(t) = e^{iH_0 t/\hbar} \\ U_1(t) \equiv U(t) U_0^\dagger(t) = U(t) e^{-iH_0 t/\hbar} \end{cases}$$

$$\langle A \rangle_t = \text{Tr} (U_1(t) U_0(t) A U_0^\dagger(t) U_1^\dagger(t) W_0)$$

$$= \text{Tr} (U_0(t) A U_0^\dagger(t) U_1^\dagger(t) W_0 U_1(t))$$

$$= \text{Tr} (A^D(t), W^D(t))$$

burada

$$A^D = U_0(t) A U_0^\dagger(t) \quad 1.51$$

$$W^D = U_1^\dagger(t) W_0 U_1(t) \quad 1.52.$$

1.22 ve 1.51. 'in benzerliğine dikkat!!

$H_0 = H$ ve $H_1 = 0 \Rightarrow U_0(t) = U(t)$ ve $U_1(t) = I$, öyleki

Dirac Temili $\rightarrow$ Heisenberg tem..

Schrödinger tem. 'de 1.15, Dirac tem. 'de 1.52'ye benzer. $[H_0, H_1] = 0 \Rightarrow$ bu nedenle de her iki de komütatör. Çünkü 0 zaman

$$U_1(t) = e^{i t H_1 / \hbar} e^{-i t H_0 / \hbar} = e^{i t H_1 / \hbar} \text{ olur.}$$

$H_0 = 0$ ve $H_1 = H$ durumunda $U_0(t) = I$ ve $U_1(t) = U(t)$ olur.

$\rightarrow$ Schrödinger tem. .

W_0 : saf bir durum $(= |\psi_0\rangle \langle \psi_0|)$

tanım : $|\psi(t)\rangle_D \equiv U_1^\dagger(t) |\psi_0\rangle$

$$W^D(t) = |\psi(t)\rangle_D \langle \psi(t)| \quad W^D(t) = U_1^\dagger(t) W_0 U_1(t)$$

$$i\hbar \frac{d}{dt} |\psi(t)\rangle_D = H_1 |\psi(t)\rangle_D$$

D.P 'da fiziksel olarak ölçülebilir nicelik $\langle A \rangle_t$ 'nin zaman bağımlılığı, gözlenirlik ve durum ve fiziksel olarak zamanla değişimi ile tanımlanır.

A gözlenirlik $|\alpha(t)\rangle_D$ örneklere

$$A^D(t) |\alpha(t)\rangle_D = a(t) |\alpha(t)\rangle_D$$

$|\alpha_0\rangle_D$ 'nin $e^{i t H_0 / \hbar}$ tarafından dönüştürülmesiyle verilir.

$$|a+\rangle_D = e^{i\pi/4} |a0\rangle_D$$

$|\psi(t)\rangle_D$ durum vektörleri $|\psi_0\rangle$ 'ın ters yönde

$U_1^\dagger(t)$ ($= e^{-iH_1 t/\hbar}$ eğer $[H_0, H_1] = 0$ ise) tarafından dönüştürülmesi ile elde edilir.

$$|\psi(t)\rangle_D = U_1^\dagger(t) |\psi_0\rangle$$

olasılıkları zaman bağımsızlığı S.P ve H.P de olduğu gibi dir. : $U(t) = U_1 U_0$

$$\begin{aligned} |\langle a+|\psi_0\rangle|^2 &= |\langle a0|U_1^\dagger(t)|\psi_0\rangle|^2 \\ &= |\langle a0|U_0^\dagger(t)U_1^\dagger(t)|\psi_0\rangle|^2 \\ &= |\langle a+|\psi(t)\rangle_D|^2 \end{aligned}$$

0 Homojen olmayan sistemler

$$\frac{dA(t)}{dt} = \frac{1}{i\hbar} [A(t), H(t)] + \frac{\partial A(t)}{\partial t} \quad 1.59 \quad \underline{\text{ave deni}}$$

$A = A(Q_i(t), P_i(t), t)$ olarak yazabiliriz

Poisson parantezleri $\rightarrow (1/i\hbar \times \text{komütasyon})$

reğisimi ile elde edilir

Surviving P .

$$W(1) = U^\dagger(1) W_0 U(1)$$

$$U(1) = e^{i\pi/t}$$

$$i\hbar \frac{d}{dt} |\psi(1)\rangle = H |\psi(1)\rangle$$

$$|\psi(1)\rangle = U^\dagger(1) |\psi_0\rangle$$

$$= e^{-i\pi t/t} |\psi_0\rangle$$

~~Surviving~~ P .

$$A(1) = U(1) A U^\dagger(1)$$

$$U(1) = e^{i\pi/t}$$

$$i\hbar \frac{d}{dt} A(1) = [A(1), H]$$

Trace P .

$$A^D(1) = U_0(1) A U_0^\dagger(1)$$

$$W^D(1) = U_1^\dagger(1) W_0 U_1(1)$$

$$U_1(1) = U(1) U_0^\dagger(1)$$

$$i\hbar \frac{d}{dt} |\psi^D\rangle = H_1 |\psi(1)\rangle_D$$

$$|\psi(1)\rangle_D = U_1^\dagger(1) |\psi_0\rangle$$

$$(\Rightarrow) [H_0, H_1] = 0 \text{ re.}$$

prb. XII.1. H , a ve $a^\dagger$ daha evvel harmonik osilatör için tanımlanmış operatörler olsun. Zaman evrimi aksiyomunu kullanarak da/dt ve $da^\dagger/dt$ 'yi hesaplayınız.

$$H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

$$i\hbar \frac{da}{dt} = [a, H] = \hbar\omega [a, a^\dagger a] = \hbar\omega a$$

$$\Rightarrow \frac{da}{dt} + i\omega a = 0 \quad \Rightarrow a(t) = a(0) e^{-i\omega t}$$

$$a^\dagger(t) = a^\dagger(0) e^{i\omega t}$$

prb. XII.2. $K(t)$ dış kuvvete maruz kalmış bir harmonik osilatör için H Hamiltoniyeni

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2} Q^2 - QK(t) = H_0 - QK(t)$$

toplam enerji

a) H 'yi a , $a^\dagger$ ve I cinsinden ifade ediniz.

$$Q = \left(\hbar / 2m\omega \right)^{1/2} (a + a^\dagger)$$

$$\Rightarrow H = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right) - \left(\frac{\hbar}{2m\omega} \right)^{1/2} (a + a^\dagger) K(t)$$

b) Zaman evrimi üzerine aktyomu kullanarak, da/dt 'yi hesaplaymız.

$$i\hbar \frac{da}{dt} = \hbar\omega a - \left(\frac{\hbar}{2m\omega}\right)^{1/2} [a, a + a^\dagger] K(t)$$

$$i \frac{da}{dt} = \omega a - \frac{1}{(2m\omega\hbar)^{1/2}} K(t)$$

$$\Rightarrow \frac{da}{dt} + i\omega a = \frac{i}{\sqrt{2m\omega\hbar}} K(t)$$

c) Dif. denk.'i çözüyoruz.

$$\begin{aligned} \frac{d}{dt} (a e^{i\omega t}) &= e^{i\omega t} \frac{da}{dt} + i\omega e^{i\omega t} a \\ &= e^{i\omega t} \left(\frac{da}{dt} + i\omega a \right) \end{aligned}$$

$$\frac{d}{dt} (a e^{i\omega t}) = \frac{i}{\sqrt{2m\omega\hbar}} K(t) e^{i\omega t}$$

$$e^{i\omega t} a = \frac{i}{\sqrt{2m\omega\hbar}} \int_0^t dt' e^{i\omega t'} K(t')$$

$$a = \frac{i}{\sqrt{2m\omega\hbar}} \int_0^t dt' e^{i\omega(t-t')} K(t')$$

Genel çözüm

$$a(t) = a(0) e^{-i\omega t} + \frac{i}{\sqrt{2m\omega\hbar}} \int_0^t dt' e^{i\omega(t'-t)} K(t')$$

Prb. XII.3. Prb. 2 a) için $\psi(t)$ bir kuvveti altındaki harmonik osilatör için $N = a^\dagger a$ 'nın ϕ_n ortonormal örnektörleri kuvvetiz durumda kullanılarak, sonunda bulunabilir:

$$\psi_n(t) = \frac{1}{\sqrt{n!}} [a^\dagger(t)]^n \phi_0(t)$$

taban dur.

$t=0$ anında harmonik osilatörün enerjisi $\hbar\omega/2$ ölçülmüştür.

Bu anı taban durumunu $\phi_0 = \psi_0(t=0)$ gösteriniz.

a) Enerji ölçümünden sonra harmonik osilatör için istatistiksel op. nedir?

$$W = \Lambda = |\phi_0\rangle\langle\phi_0|$$

b) Bir t zaman sonra $E = \hbar\omega(n+1/2)$ enerjili bir değeri ölçmenin olasılığını $W_n = |\langle\phi_n, \phi_0\rangle|^2$ olarak gösteriniz.

$$W' = \sum_n \Lambda_n W \Lambda_n$$

0

$$\Lambda_n = |nt\rangle\langle nt| \quad , \quad |nt\rangle = |\phi_n(t)\rangle$$

$$W' = \sum_n |nt\rangle\langle nt| \underbrace{|\phi_0\rangle\langle\phi_0|}_{\text{taban dur.}} |nt\rangle\langle nt|$$

$$= \sum_n |\langle nt|\phi_0\rangle|^2 |nt\rangle\langle nt|$$

$$= \sum_n W_n |nt\rangle\langle nt|$$

$$c) \quad w_n = \frac{|F(t)|^{2n}}{n!} e^{-|F(t)|^2}$$

olayın gösteriniz, burada, $F(t) = \frac{i}{\sqrt{2m\omega}} \int_0^t K(z) e^{i\omega z} dz$

$$|nt\rangle = C_1 \frac{1}{\sqrt{n!}} [a^\dagger(t)]^n |\phi_0\rangle$$

$$= C_1 \frac{1}{\sqrt{n!}} [a^\dagger(t)]^n e^{-i\omega t/\hbar} |00\rangle \quad | \phi_0 \rangle$$

$$= C_2 e^{-i\omega t} \frac{1}{\sqrt{n!}} [a^\dagger(t)]^n |\phi_0\rangle$$

$$\langle \phi_0 | nt \rangle = C_2 e^{-i\omega t} \frac{1}{\sqrt{n!}} \langle \phi_0 | a^\dagger(t)^n | \phi_0 \rangle$$

$$a^\dagger(t) = a_0^\dagger e^{i\omega t} + e^{i\omega t} F(t)$$

$$\langle \phi_0 | nt \rangle = \frac{C e^{-i\omega t}}{\sqrt{n!}} \langle \phi_0 | [a^\dagger(0) e^{i\omega t} + e^{i\omega t} F(t)]^n | \phi_0 \rangle$$

$$= \frac{C}{\sqrt{n!}} e^{-i\omega(1-\eta)t} \langle \phi_0 | [a^\dagger(0) + F(t)]^n | \phi_0 \rangle$$

$$w_n = |\langle \phi_0 | nt \rangle|^2, \quad \sum_n w_n = 1$$

$$1 = |C|^2 \sum_n \frac{(|F|^2)^n}{n!} = |C|^2 e^{|F|^2}$$

$$|C|^2 = e^{-|F|^2}$$

$$w_n = e^{-|F|^2} \frac{(|F|^2)^n}{n!}$$

○ d) $t=0$ daki enerji ölçümünden hemen sonra konum op. için beklenen değeri

$$\langle Q \rangle = \frac{1}{m\omega} \int_0^t k(\tau) \sin(\omega - \tau) d\tau$$

$\omega(t-\tau)$

olduğunu gösteriniz.

$$Q = \left(\hbar / 2m\omega \right)^{1/2} (a + a^\dagger) \quad a \equiv a(t) \quad a^\dagger \equiv a^\dagger(t)$$

$$0 \langle \phi_0 | Q | \phi_0 \rangle = \left(\frac{\hbar}{2m\omega} \right)^{1/2} [\langle \phi_0 | (a + a^\dagger) | \phi_0 \rangle]$$

$$a(t) = a(0) e^{-i\omega t} + e^{-i\omega t} F(t)$$

$$a^\dagger(t) = a^\dagger(0) e^{i\omega t} + e^{i\omega t} \bar{F}(t)$$

$$\langle Q \rangle = \left(\hbar / 2m\omega \right)^{1/2} [e^{-i\omega t} F(t) + e^{i\omega t} \bar{F}(t)]$$

$$= \left(\frac{\hbar}{2m\omega} \right)^{1/2} \frac{i}{\sqrt{2m\omega\hbar}} \left\{ \int_0^t k(\tau) e^{i\omega(\tau-t)} d\tau + \int_0^t k(\tau) e^{-i\omega(\tau-t)} d\tau \right\}$$

$$XII.4. \quad K(t) = \begin{cases} K(t) & t > 0 \\ 0 & t < 0 \end{cases}$$

Şeklindeki bir dış kuvvetin etriği altındaki harmonik osilatör $t=0$ anında $\hbar\omega/2$ enerjili durumda'dır. Enerji'nin t anında $\hbar\omega(m+1/2)$ 'ye eşit ya da daha düşük olma olasılığı $\Lambda = \sum_{n=0}^m \Lambda_{\psi_n(t)}$ ile karakterize edilir, burada $\Lambda_{\psi_n(t)}$ $\psi_n(t)$ tarafından verilen 1 boyutlu alt uzaylar üzerinde projeksiyon op.'leridir. (a) Enerjiyi $\hbar\omega(m+1/2)$ ye eşit ya da daha düşük bulmanın olasılığı nedir?

$$\circ \quad \langle nt | nt \rangle = 1 \Rightarrow |c|^2 = \frac{1}{\sum_{n=0}^m \frac{1}{n!} |F(t)|^{2n}}$$

$$\text{Tr}(\Lambda) = \langle nt | \Lambda | nt \rangle = 1$$

b) Λ 'nın pozitif sonuçlu bir ölçümünden sonra W' istatistiksel op.'ü nedir?

$$\circ \quad W' = \Lambda_n - \Lambda \Lambda_n = \sum_{n=0}^m \Lambda_n - \Lambda \Lambda_n$$

$$= \sum_{n'=0}^m \sum_{n=0}^m |n't\rangle \langle n't| (|nt\rangle \langle nt|) |n't\rangle \langle n't|$$

$$= \sum_{n'=0}^m |n't\rangle \langle n't|$$

c)

$$w' \psi_{n'} = \begin{cases} 0 & n' > n \\ = \frac{1}{\sum_{n=0}^{\infty} (1/n!) |F(t)|^2} \sum_{n=0}^{\infty} |\psi(t)\rangle e^{i(n'-n)\omega t} \\ \times \frac{1}{\sqrt{n!n'}} [F(t)]^n [\bar{F}(t)]^{n'} & n' \leq n \end{cases}$$

darüberhin gestrichelt.

$$\begin{aligned} w' \psi_{n'} &= w' |n't\rangle \\ &= \left(\sum_{n=0}^{\infty} |nt\rangle \langle nt| \right) |n't\rangle \end{aligned}$$

$$|nt\rangle = C \frac{1}{\sqrt{n!}} e^{i\omega t(n+1/2)} a^\dagger^n |00\rangle$$

$$|n't\rangle = C \frac{1}{\sqrt{n'!}} e^{-i\omega t(n'+1/2)} (a^\dagger)^{n'} |00\rangle$$

$$\langle nt | n't \rangle = |C|^2 \frac{1}{\sqrt{n!n'}} e^{i\omega(n'-n)t} \langle 00 | a^n (a^\dagger)^{n'} | 00 \rangle$$

$$\begin{aligned} & \langle 00 | \left(a(0) e^{-i\omega t} + e^{-i\omega t} F(t) \right)^n \left(a^\dagger(0) e^{i\omega t} + e^{i\omega t} \bar{F}(t) \right)^{n'} | 00 \rangle \\ &= |C|^2 \frac{e^{i\omega(n'-n)t}}{\sqrt{n!n'}} (F(t))^n (\bar{F}(t))^{n'} \end{aligned}$$

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