BÖLÜM 13

ELEKTRONLARIN ELEKTROMAGNETİK ALAN İLE ETKİLEŞMESİ

Maxwell Denklemleri :

$\vec{\nabla}\cdot\vec{B}(\vec{r},t)=0$ magnetik monopol yokluğu

$\vec{\nabla}\cdot\vec{E}(\vec{r},t)=4\pi\rho(\vec{r},t)$ Coulomb yasası

$\vec{\nabla}\times\vec{E}(\vec{r},t)+\frac{1}{c}\frac{\partial\vec{B}(\vec{r},t)}{\partial t}=0$ Faraday yasası

$\vec{\nabla}\times\vec{B}(\vec{r},t)-\frac{1}{c}\frac{\partial\vec{E}(\vec{r},t)}{\partial t}=\frac{4\pi}{c}\vec{J}(\vec{r},t)$ Ampère yasası

Yük korunumu

$$\frac{\partial\rho(\vec{r},t)}{\partial t}+\vec{\nabla}\cdot\vec{J}(\vec{r},t)=0$$

$$\vec{B}(\vec{r},t)=\vec{\nabla}\times\vec{A}(\vec{r},t)$$

$$\vec{E}(\vec{r},t)=-\vec{\nabla}\phi-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$f(\vec{r},t)$ keyfi bir fonk. olmak üzere

$$\left.\begin{aligned}\vec{A}'&=\vec{A}-\vec{\nabla}f\\ \phi'&=\phi+\frac{1}{c}\frac{\partial f}{\partial t}\end{aligned}\right\}\text{ Aynı } \vec{E} \text{ ve } \vec{B} \text{ alanlarını verirler.}$$

$(\vec{A},\phi)\longrightarrow(\vec{A}',\phi')$ ayar dönüşümü

BS K

(*) $\vec{\nabla}\cdot\vec{E} = 4\pi\rho$ idi. Coulomb Torsen

$$\vec{\nabla}\cdot\left(-\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi\right) = 4\pi\rho$$

$$\boxed{-\vec{\nabla}^2\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = 4\pi\rho}$$

(*) $\vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$

$$\Rightarrow \underbrace{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)}_{\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \vec{\nabla}^2\vec{A}} - \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi\right) = \frac{4\pi}{c}\vec{J}$$

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \vec{\nabla}^2\vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} + \frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\phi\right) = \frac{4\pi}{c}\vec{J}$$

$$\boxed{-\vec{\nabla}^2\vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} + \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = \frac{4\pi}{c}\vec{J}}$$

Yük yoğunluğu zamandan bağımsız ise, $\vec{\nabla}\cdot\vec{A}$ seçimi yapılır (Coulomb Ayarı) — durgun

$$\Rightarrow \vec{\nabla}^2\phi = -4\pi\rho$$

zamandan bağımsız ϕ

$$\Rightarrow -\vec{\nabla}^2\vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{4\pi}{c}\vec{J}$$

BS K

Yük yoğunluğu durgun olduğu zaman

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0 \qquad \text{Lorentz ayarı}$$

$$-\vec{\nabla}^2 \phi - \frac{1}{c}\frac{\partial}{\partial t}\left(-\frac{1}{c}\frac{\partial \phi}{\partial t}\right) = 4\pi\rho$$

$$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -4\pi\rho$$

$$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{J}$$

$$\Box_\mu^2 A_\mu = -\frac{4\pi}{c} J_\mu \qquad A_\mu = (A_0, A_i) \quad J_\mu = (c\rho, J_i)$$

Q.M.'e geçiş Lorentz denkleminin Hamilton formülasyonunu gerektirir. Etkileşimin yokluğunda

$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} \qquad H = \frac{p^2}{2\mu} + V(q)$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

$$\Rightarrow \mu \frac{d^2 x_i}{dt^2} = -\frac{\partial V}{\partial q_i} = F_i$$

Etkileşimin varlığında,

$$H = \frac{\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2}{2\mu} + e\phi$$

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$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} = \frac{p_i + (e/c)A_i}{\mu}$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i} = -\frac{1}{2\mu}\frac{\partial}{\partial x_i}\left(p_k^2 + \frac{e^2}{c^2}A_k^2 + 2\frac{e}{c}p_k A_k\right) + e\frac{\partial \phi}{\partial x_i}$$

$$= -\frac{1}{\cancel{2}\mu}\left\{\cancel{2}\frac{e^2}{c^2}A_k\frac{\partial A_k}{\partial x_i} + \cancel{2}\frac{e}{c}p_k\frac{\partial A_k}{\partial x_i}\right\} + e\frac{\partial \phi}{\partial x_i}$$

$$= -\frac{e}{\mu c}\underbrace{\left(p_k + \frac{e}{c}A_k\right)}_{\mu\, dx_k/dt}\frac{\partial A_k}{\partial x_i} + e\frac{\partial \phi}{\partial x_i}$$

$$\Rightarrow \mu\frac{d^2x_i}{dt^2} = \frac{dp_i}{dt} + \frac{e}{c}\frac{dA_i}{dt}$$

$$= \frac{dp_i}{dt} + \frac{e}{c}\left(\frac{\partial A_i}{\partial t} + \frac{\partial A_i}{\partial x_k}\frac{dx_k}{dt}\right)$$

$$= \underbrace{e\frac{\partial \phi}{\partial x_i} + \frac{e}{c}\frac{\partial A_i}{\partial t}}_{-eE_i} + \underbrace{\frac{e}{c}\frac{\partial A_i}{\partial x_k}\frac{dx_k}{dt} - \frac{e}{c}\frac{dx_k}{dt}\frac{\partial A_k}{\partial x_i}}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}\ , \quad (\vec{v}\times\vec{B})_i\left(-e/c\right)$$

$$B_1 = (\vec{\nabla}\times\vec{A})_1 = \partial_2 A_3 - \partial_3 A_2$$

$$(\vec{v}\times\vec{B})_1 = v_2 B_3 - v_3 B_2$$

$$= v_2(\partial_1 A_2 - \partial_2 A_1) - v_3(\partial_3 A_1 - \partial_1 A_3)$$

BS K

Bir elektromagnetik alandaki bir elektron için Schrödinger Denklemi

$$\left[\frac{\left((\hbar/i)\vec{\nabla} + (e/c)\vec{A}\right)^2}{2\mu} + e\,\phi(\vec{r},t)\right]\psi(\vec{r},t) = i\hbar\frac{\partial\psi}{\partial t}$$

$$\vec{A}' = \vec{A} - \vec{\nabla} f \qquad \phi' = \phi + \frac{1}{c}\frac{\partial f}{\partial t}$$

$$\left\{\frac{\left[\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}' + \frac{e}{c}\vec{\nabla} f\right]^2}{2\mu} + e\phi' - \frac{1}{c}e\frac{\partial f}{\partial t}\right\}\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\psi' = e^{i\Lambda}\psi$$

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial t}\left(e^{-i\Lambda}\psi'\right) = -i\frac{\partial\Lambda}{\partial t}\psi + e^{-i\Lambda}\frac{\partial\psi'}{\partial t}$$

$$= -i\,e^{-i\Lambda}\psi'\frac{\partial\Lambda}{\partial t} + e^{-i\Lambda}\frac{\partial\psi'}{\partial t}$$

$$\frac{\hbar}{i}\vec{\nabla}\psi = \frac{\hbar}{i}\vec{\nabla}\left(e^{-i\Lambda}\psi'\right) = -\hbar(\vec{\nabla}\Lambda)e^{-i\Lambda}\psi'$$

$$+\,e^{-i\Lambda}\frac{\hbar}{i}\vec{\nabla}\psi'$$

$$\left\{\frac{1}{2\mu}\left[\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}' + \frac{e}{c}\vec{\nabla} f\right]\left[+\frac{\hbar}{i}\cancel{e^{-i\Lambda}}\vec{\nabla}\psi' - \hbar\cancel{e^{-i\Lambda}}(\vec{\nabla}\Lambda)\psi' + \frac{e}{c}\vec{A}'\cancel{e^{-i\Lambda}}\psi' + \frac{e}{c}\vec{\nabla} f\cancel{e^{-i\Lambda}}\psi'\right]\right.$$

$$\left. + \left(e\phi' - \frac{e}{c}\frac{\partial f}{\partial t}\right)\cancel{e^{-i\Lambda}}\psi' + i\cancel{e^{-i\Lambda}}\cancel{\hbar}\psi'\frac{\partial\Lambda}{\partial t}\right\} = \cancel{e^{-i\Lambda}}\cancel{\hbar}\frac{\partial\psi'}{\partial t}$$

$$\Lambda = (e/\hbar c)\,f$$

BS K

$$\left[\frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla}+\frac{e}{c}\vec{A}\right)\cdot\left(\frac{\hbar}{i}\vec{\nabla}+\frac{e}{c}\vec{A}\right)+e\phi(\vec{r})\right]\psi(\vec{r})=E\psi(\vec{r})$$

$\vec{A}=\vec{A}(\vec{r})$ ve $\phi=\phi(\vec{r})$ alındı. Dolayısı ile

$$\psi(\vec{r},t)=e^{-iEt/\hbar}\psi(\vec{r})$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi-\frac{ie\hbar}{2\mu c}\vec{A}\cdot\vec{\nabla}\psi-\frac{ie\hbar}{2\mu c}\underbrace{(\vec{\nabla}\cdot\vec{A}\psi)}_{\cancel{(\vec{\nabla}\cdot\vec{A})}\psi+\vec{A}\cdot(\vec{\nabla}\psi)}+\frac{e^2}{2\mu c^2}A^2\psi=(E-e\phi)\psi$$

C. ayar.

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi-\frac{ie\hbar}{2\mu c}\vec{A}\cdot\vec{\nabla}\psi-\frac{ie\hbar}{\mu c}\vec{A}\cdot(\vec{\nabla}\psi)+\frac{e^2}{2\mu c^2}A^2\psi=(E-e\phi)\psi$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi-\frac{ie\hbar}{\mu c}\vec{A}\cdot\vec{\nabla}\psi+\frac{e^2}{2\mu c^2}A^2\psi=(E-e\phi)\psi$$

Sabit Magnetik alan:

$$\vec{A}=-\frac{1}{2}\vec{r}\times\vec{B} \qquad \vec{B}=B\hat{z}$$

$$=-\frac{1}{2}\vec{r}\times B\hat{z}=-\frac{B}{2}\left(-x\hat{y}+y\hat{x}\right)$$

$$=-B/2\,(y,-x,0)$$

$$\vec{\nabla}\times\vec{A}=\vec{B}$$

BS K

$$\frac{ie\hbar}{\mu c}\vec{A}\cdot\vec{\nabla}\psi = -\frac{ie\hbar}{\mu c}\frac{1}{2}(\vec{r}\times\vec{B})\cdot(\vec{\nabla}\psi)$$

$$= -\frac{ie\hbar}{2\mu c}\vec{B}\cdot(\vec{r}\times\vec{\nabla})\psi$$

$$= \frac{e}{2\mu c}\vec{B}\cdot(\underbrace{\vec{r}\times\frac{\hbar}{i}\vec{\nabla}}_{\vec{L}})\psi = \frac{e}{2\mu c}\vec{B}\cdot\vec{L}\psi$$

$$= \frac{e}{2\mu c}B L_z\psi$$

$$\frac{e^2}{2\mu c^2}\frac{1}{4}(\vec{r}\times\vec{B})^2\psi = \frac{e^2}{8\mu c^2}(r^2B^2\sin^2\theta)\psi$$

$$= \frac{e^2}{8\mu c^2}r^2B^2(1-\cos^2\theta)\psi = \frac{e^2}{8\mu c^2}(r^2B^2 - \underbrace{(\vec{r}\cdot\vec{B})^2}_{z^2B^2})\psi$$

$$= \frac{e^2B^2}{8\mu c^2}(r^2-z^2)\psi = \frac{e^2B^2}{8\mu c^2}(x^2+y^2)\psi$$

$$\frac{(e^2/8\mu c^2)a_0^2B^2}{(e/2\mu c)B\hbar} \simeq B/(9\times10^9\ \text{Gauss})$$

$B \lesssim 10^4$ Gauss

thermal collectibility

$$\frac{(e/2\mu c)B\hbar}{(e^2/a_0)} \simeq B/(5\times10^9\ \text{Gauss})$$

BS K

Normal Zeeman Olayı:

Sadece lineer terimi ele alalım

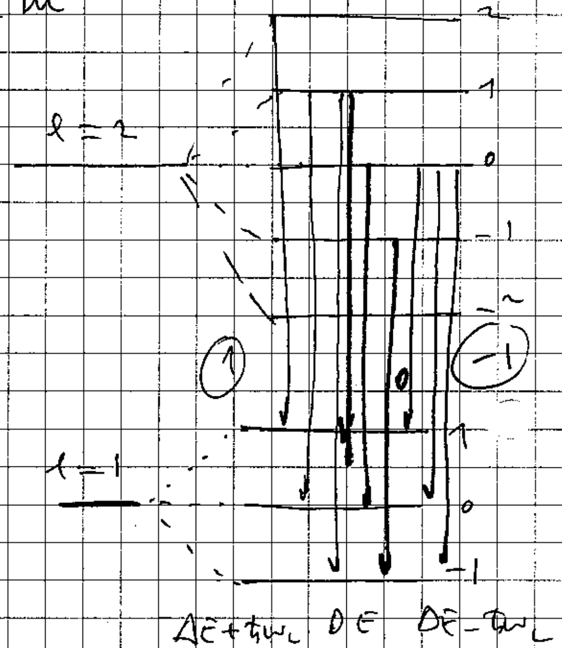
$$H_1 = \frac{e}{2\mu c} B L_z = \omega_L L_z$$

Larmor.

L^2, L_z eşzamanlı ölçüm

$$H_1 u_{n\ell m} = \hbar \omega_L m u_{n\ell m}$$

$$E = -\frac{1}{2}\mu c^2 \frac{(Z\alpha)^2}{n^2} + \hbar\omega_L m$$



Yarılmanın büyüklüğü

$$\hbar\omega_L = \frac{e\hbar}{2\mu c}\left(\frac{B}{e/a_0^2}\right)\frac{e}{a_0^2}$$

$$= \frac{e^2\hbar}{2\mu c}\left(\frac{\mu c \alpha}{\hbar}\right)^2 \frac{B}{e/a_0^2}$$

$$= \left(\frac{1}{2}\alpha^2\mu c^2\right)\alpha \frac{B}{e/a_0^2} = \left(\frac{B}{2.4\times 10^9}\right)\times 13.6\ \text{eV}.$$

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + \frac{eB}{2\mu c}L_z\psi + \frac{e^2B^2}{8\mu c^2}(x^2+y^2)\psi = E\psi$$

$$x=\rho\cos\varphi \quad y=\rho\sin\varphi \quad z=z$$

$$\rho=\sqrt{x^2+y^2} \quad \varphi=tg^{-1}\frac{y}{x}$$

$$\frac{\partial}{\partial x}=\frac{\partial\rho}{\partial x}\frac{\partial}{\partial\rho}+\frac{\partial\varphi}{\partial x}\frac{\partial}{\partial\varphi}+\frac{\partial z}{\partial x}\frac{\partial}{\partial z}$$

$$\frac{\partial\rho}{\partial x}=\frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cancel{2}x=\frac{1}{\cancel{\rho}}\cancel{\rho}\cos\varphi$$

$$\frac{\partial\varphi}{\partial x}=\frac{1}{1+\frac{y^2}{x^2}}\cdot\left(\frac{-y}{x^2}\right)=-\frac{1}{\rho^2}\rho\sin\varphi=-\frac{\sin\varphi}{\rho}$$

$$\frac{\partial z}{\partial x}=0$$

$$\boxed{\frac{\partial}{\partial x}=\cos\varphi\frac{\partial}{\partial\rho}-\frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}+0}$$

$$\frac{\partial\rho}{\partial y}=\frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cancel{2}y=\frac{1}{\rho}\rho\sin\varphi=\sin\varphi$$

$$\frac{\partial\varphi}{\partial y}=\frac{1}{1+y^2/x^2}\frac{1}{x}=\frac{1}{\rho^2}\rho\cos\varphi=\frac{\cos\varphi}{\rho}$$

$$\frac{\partial z}{\partial y}=0$$

$$\boxed{\frac{\partial}{\partial y}=\sin\varphi\frac{\partial}{\partial\rho}+\frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}}$$

$$\boxed{\frac{\partial}{\partial z}=\frac{\partial}{\partial z}}$$

BS K

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$$= \cos^2\varphi\frac{\partial^2}{\partial\rho^2} - \sin\varphi\cos\varphi\frac{\partial}{\partial\varphi}\frac{\partial}{\partial\rho}\frac{1}{\rho}$$

$$- \frac{\sin\varphi}{\rho}\partial_\rho\partial_\varphi\cos\varphi + \frac{\sin\varphi}{\rho^2}\frac{\partial}{\partial\varphi}\left(\sin\varphi\frac{\partial}{\partial\varphi}\right)$$

$$= \cos^2\varphi\,\partial_\rho^2 - \sin\varphi\cos\varphi\,\partial_\varphi\left(-\frac{1}{\rho^2}\right)$$

$$+ \sin^2\varphi\,\cancel{\cos\varphi}\,\frac{1}{\rho}\,\partial_{\varphi\rho} + \frac{\sin\varphi\cos\varphi}{\rho^2}\partial_\varphi + \frac{\sin^2\varphi}{\rho^2}\partial^2_\varphi$$

$$\partial_y^2 = \left(\sin\varphi\,\partial_\rho + \frac{\cos\varphi}{\rho}\partial_\varphi\right)\left(\sin\varphi\,\partial_\rho + \frac{\cos\varphi}{\rho}\partial_\varphi\right)$$

$$= \sin^2\varphi\,\partial^2_\rho + \sin\varphi\cos\varphi\,\partial_\varphi\left(-1/\rho^2\right)$$

$$+ \frac{\cos^2\varphi}{\rho}\partial_\rho + \frac{\cos\varphi(-\sin\varphi)}{\rho^2}\partial_\varphi + \frac{\cos^2\varphi}{\rho^2}\partial^2_\varphi$$

$$\nabla^2 = \partial_\rho^2 + \frac{1}{\rho}\partial_\rho + \frac{1}{\rho^2}\partial^2_\varphi + \partial^2_z$$

$$= \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$L_z = xp_y - yp_x = \frac{\hbar}{i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$= \frac{\hbar}{i}\left(\cancel{\rho}\cos\varphi\left(\cancel{\sin\varphi}\frac{\partial}{\partial\rho} + \frac{\cos\varphi}{\cancel{\rho}}\frac{\partial}{\partial\varphi}\right)\right.$$

$$\left. - \cancel{\rho}\sin\varphi\left(\cancel{\cos\varphi}\frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\cancel{\rho}}\frac{\partial}{\partial\varphi}\right)\right)$$

BS K $= (\hbar/i)\ \partial/\partial\varphi$

$$\left\{-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}\right)\right.$$

$$\left.+\omega_L\frac{\hbar}{i}\frac{\partial}{\partial\varphi}+\frac{1}{2}\mu\omega_L^2\rho^2-E\right\}\psi(\rho,\varphi,z)=0$$

$$\left[\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}\right)\right.$$

$$\left.-\frac{2\mu}{\hbar^2}\left[\omega_L\frac{\hbar}{i}\frac{\partial}{\partial\varphi}+\frac{1}{2}\mu\omega_L^2\rho^2-E\right]\right]\psi=0$$

$$\psi(\vec{r})=U(\rho)\Phi(\varphi)Z(z)$$

$$\left[\!\left[\Phi Z\frac{d^2U}{d\rho^2}+\frac{1}{\rho}\Phi Z\frac{dU}{d\rho}+\frac{1}{\rho^2}UZ\frac{d^2\Phi}{d\varphi^2}+U\Phi\frac{d^2Z}{dz^2}\right.\right.$$

$$\left.\left.+\frac{2\mu}{\hbar^2}i\hbar\omega_L UZ\frac{d\Phi}{d\varphi}-\frac{2\mu}{\hbar^2}U\Phi Z\left[\frac{1}{2}\mu\omega_L^2\rho^2-E\right]\right]\!\right]=0$$

$$\left\{\frac{1}{U}\frac{d^2U}{d\rho^2}+\frac{1}{\rho U}\frac{dU}{d\rho}+\frac{1}{\rho^2\Phi}\frac{d^2\Phi}{d\varphi^2}+\frac{1}{Z}\frac{d^2Z}{dz^2}\right.$$

$$\left.+\frac{2\mu i\hbar\omega_L}{\hbar^2}\frac{1}{\Phi}\frac{d\Phi}{d\varphi}-\frac{2\mu}{\hbar^2}\left[\frac{1}{2}\mu\omega_L^2\rho^2-E\right]\right\}=0$$

$$\frac{1}{U}\frac{d^2U}{d\rho^2}+\frac{1}{\rho U}\frac{dU}{d\rho}+\frac{1}{\rho^2\Phi}\frac{d^2\Phi}{d\varphi^2}+\frac{2\mu i\hbar\omega_L}{\hbar^2}\frac{1}{\Phi}\frac{d\Phi}{d\varphi}$$

$$-\frac{2\mu}{\hbar^2}\left[\frac{1}{2}\mu\omega_L^2\rho^2-E\right]=-\frac{1}{Z}\frac{d^2Z}{dz^2}=k^2$$

BS K

$$\frac{d^2 Z}{dz^2} + k^2 Z = 0 \Rightarrow Z(z) = e^{\pm ikz}$$

$$L_z \Phi = m\hbar\Phi \Rightarrow \frac{\cancel{\hbar}}{i}\frac{d\Phi}{d\varphi} = m\cancel{\hbar}\Phi$$

$$\frac{d\Phi}{\Phi} = im\,d\varphi \Rightarrow \Phi(\varphi) = e^{im\varphi}$$

$$\frac{1}{u}\frac{d^2u}{d\rho^2} + \frac{1}{\rho u}\frac{du}{d\rho} + \frac{1}{\rho^2}(-m^2) + \frac{2\mu i\hbar\omega_L}{\hbar^2}\,im$$

$$-\frac{2\mu}{\hbar^2}\left[\frac{1}{2}\mu\omega_L^2\rho^2 - E\right] - k^2 = 0$$

$$\frac{d^2u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} - \frac{m^2}{\rho^2}u - \frac{2\mu\hbar\omega_L}{\hbar^2}mu \qquad \omega_L = \frac{eB/\mu c}{2}$$

$$-\frac{2\mu}{\hbar^2}\left[\frac{1}{2}\mu\omega_L^2\rho^2 - E\right]u - k^2u = 0$$

$$\frac{d^2u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} - \frac{m^2}{\rho^2}u - \left(\frac{\mu\omega_L}{\hbar}\right)^2\rho^2 u$$

$$+\left(\frac{2\mu E}{\hbar^2} - \frac{2\mu\omega_L}{\hbar}m - k^2\right)u = 0$$

$$x = \sqrt{\frac{\mu\omega_L}{\hbar}}\,\rho = \sqrt{\frac{\cancel{\mu} eB}{\hbar 2\cancel{\mu} c}}\,\rho = \sqrt{\frac{eB}{2\hbar c}}\,\rho.$$

BS K

$$\frac{d^2u}{d\rho^2}+\frac{1}{\rho}\frac{du}{d\rho}-\frac{m^2}{\rho^2}u-\left(\frac{\mu\omega_L}{\hbar}\right)^2\rho^2u$$

$$+\left(\frac{2\mu E}{\hbar^2}-\frac{2\mu\omega_L}{\hbar}m-k^2\right)u=0$$

$$\frac{d}{d\rho}=\frac{dx}{d\rho}\frac{d}{dx}=\sqrt{\frac{\mu\omega_L}{\hbar}}\frac{d}{dx} \qquad x=\sqrt{\frac{\mu\omega_L}{\hbar}}\,\rho$$

$$\frac{d^2}{d\rho^2}=\frac{\mu\omega_L}{\hbar}\frac{d^2}{dx^2}$$

$$\frac{\mu\omega_L}{\hbar}\frac{d^2u}{dx^2}+\sqrt{\frac{\mu\omega_L}{\hbar}}\frac{1}{x}\sqrt{\frac{\mu\omega_L}{\hbar}}\frac{du}{dx}-m^2\frac{\mu\omega_L}{\hbar}\frac{1}{x^2}$$

$$-\left(\frac{\mu\omega_L}{\hbar}\right)^2\cdot\frac{\hbar}{\mu\omega_L}x^2u$$

$$+\underbrace{\left(\frac{2\mu E}{\hbar^2}-\frac{2\mu\omega_L}{\hbar}m-k^2\right)}u=0 \qquad \times\ \hbar/\mu\omega_L=\lambda$$

$$\frac{d^2u}{dx^2}+\frac{1}{x}\frac{du}{dx}-\frac{m^2}{x^2}u-x^2u+\lambda u=0$$

$$\lambda=\frac{2\mu E}{\hbar^2}-\frac{2\mu\omega_L}{\hbar}m-k^2$$

Sonsuzdaki davranış:

$$\frac{d^2u}{dx^2}-x^2u\simeq 0 \rightarrow u(x)\sim e^{-x^2/2}$$

$x=0$ civarındaki davranış:

$$\frac{d^2u}{dx^2}+\frac{1}{x}\frac{du}{dx}-\frac{m^2}{x^2}\simeq 0 \rightarrow u(x)\sim x^{|m|}$$

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$$u(x) = x^{|m|} e^{-x^2/2} G(x)$$

$$\frac{d^2G}{dx^2} + \left(\frac{2|m|+1}{x} - 2x\right)\frac{dG}{dx} + \left(\lambda - 2 - 2|m|\right) G = 0$$

$$y = x^2$$

$$\frac{d^2G}{dy^2} + \left(\frac{|m|+1}{y} - 1\right)\frac{dG}{dy} + \overbrace{\frac{\lambda - 2 - 2|m|}{4y}}^{n_r} G = 0$$

$$G = L_{n_r}^{|m|}(y)$$

$$\psi(\vec{r}) = N_{n_r m}\, e^{im\varphi} e^{ikz} \left(\sqrt{\frac{\mu\omega_L}{\hbar}}\rho\right)^{|m|} e^{-\mu\omega_L\rho^2/2\hbar}$$

$$\times L_{n_r}^{|m|}\left(\frac{\mu\omega_L}{\hbar}\rho^2\right)$$

$$\frac{\lambda}{4} - \frac{1+|m|}{2} = n_r$$

$$\lambda = 4n_r + 2(1+|m|) \qquad \omega_L = \frac{eB}{2\mu c}$$

$$= \left(\frac{2\mu E}{\hbar^2} - \frac{2\mu\omega_L}{\hbar}m - k^2\right)\frac{\hbar}{\mu\omega_L}$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2\mu} + \frac{\cancel{\hbar^2}}{\cancel{2\mu}}\frac{\cancel{2\mu}}{\cancel{\hbar}}\omega_L m + \frac{\hbar\cancel{\mu}\omega_L}{2\cancel{\mu}}\left(4n_r + 2(1+|m|)\right)$$

$$= \frac{\hbar^2 k^2}{2\mu}$$

BS K

$$E = \frac{\hbar^2 k^2}{2\mu} + 2\hbar\omega_L\left(n_r + \frac{1+|m|-m}{2}\right)$$

Klasik limit

$$\vec{v} = \frac{\vec{p} + \frac{e}{c}\vec{A}}{\mu}$$

$$\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B}$$

$$\mu\,\vec{r}\times\vec{v} = \vec{r}\times\vec{p} + \frac{e}{c}\vec{r}\times\left(-\frac{1}{2}\vec{r}\times\vec{B}\right)$$

$$= \vec{L} - \frac{e}{2c}\vec{r}\times(\vec{r}\times\vec{B})$$

$$= \vec{L} - \frac{e}{2c}\left[\underbrace{\vec{r}}_{x\hat{x}+y\hat{y}+z\hat{z}}\underbrace{(\vec{r}\cdot\vec{B})}_{zB} - \underbrace{r^2\vec{B}}_{r^2 B}\right]$$

$$\bar{a}\times(\bar{b}\times\bar{c}) = b(\bar{a}\cdot\bar{c}) - c(\bar{a}\cdot\bar{b})$$

$$\mu(\vec{r}\times\vec{v})_z = L_z + \frac{eB}{2c}(x^2+y^2)$$

$$x v_y - y v_x$$

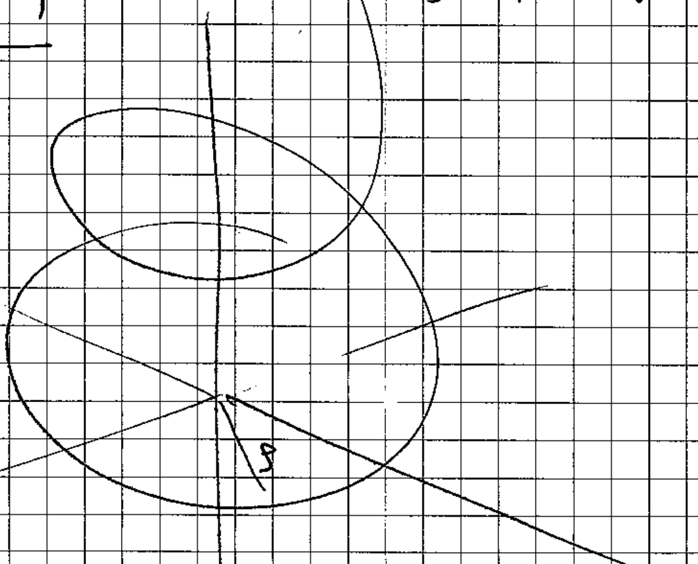
$x = \rho\cos\varphi \qquad v_x = -v\sin\varphi$

$y = \rho\sin\varphi \qquad v_y = v\cos\varphi$

$$\boxed{\mu\rho v = L_z + \frac{eB}{2c}\rho^2}$$

$$\vec{F} = -\frac{e}{c}\vec{v}\times\vec{B}$$

$$\boxed{\mu\frac{v^2}{\rho} = \frac{evB}{c}}$$ Dairesel.



$$\cancel{\frac{1}{2}\mu v^2 = \frac{evB}{2c}\rho}$$

$$\cancel{= \frac{eB}{2c\mu}\frac{\mu\rho v}{2} = \left(\frac{eB}{4c}\right)\left(\frac{L_z}{2} + \frac{eB}{4c}\rho^2\right)}$$

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$$\frac{1}{2}\mu v^2 = \frac{eBv\rho}{2c} = \frac{eB}{2c\mu}\left(L_z + \frac{eB}{2c}\rho^2\right)$$

$$= \frac{eB}{2c\mu}L_z + \left(\frac{eB}{2c}\right)^2\frac{\rho^2}{\mu} \qquad \frac{eB\rho}{c} = \mu v$$

$$\frac{eB\rho}{c} = \mu v \Rightarrow \mu v\rho = \frac{eB}{c}\rho^2$$

$$\frac{eB}{c}\rho^2 = L_z + \frac{eB}{2c}\rho^2 \Rightarrow L_z = \frac{eB\rho^2}{2c} \Rightarrow \rho = \left(\frac{2cL_z}{eB}\right)^{1/2}$$

$$\frac{1}{\cancel{2}}\mu v^2\,\frac{\cancel{2}c\mu}{eB} = L_z + \frac{eB}{2c}\rho^2$$

$$\frac{c}{eB\rho^2}\,\mu^2 v^2\rho^2 = \frac{c}{\cancel{eB\rho^2}}\,\cancel{\left(\frac{eB}{c}\right)^2}\rho^4$$

BS K

LANDAU DÜZEYLERİ

$$\vec{A} = \left(-\frac{yB}{2}, \frac{xB}{2}, 0\right) \longrightarrow \begin{array}{ll} (0, Bx, 0) & (i) \\ (-By, 0, 0) & (ii) \end{array}$$

$$= (0, Bx, 0) - (\partial_x, \partial_y, \partial_z)\left(\frac{yxB}{2}\right) \quad (i)$$

$$\frac{1}{2\mu}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 = \frac{1}{2\mu}\left(p_x^2 + \left(p_y + \frac{eB}{c}x\right)^2 + p_z^2\right)$$

$$[H, y] = 0, \quad [H, z] = 0 \qquad (i)$$

$$p_y, p_z \rightarrow \text{Sbt.}$$

$$\psi(\vec{r}) = e^{ik_y y + ik_z z}\, \Phi(x)$$

$$\frac{1}{2\mu}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 = \frac{1}{2\mu}\left(\left(p_x + \frac{eB}{c}y\right)^2 + p_y^2 + p_z^2\right)$$

$$[H, x] = 0, \quad [H, z] = 0 \qquad (ii)$$

$$p_x, p_z \rightarrow \text{Sbt.}$$

BS K

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(ii) $$\psi = e^{ik_x x + ik_z z}\,\Phi(y)$$

$$\left(\frac{\hbar}{i}\partial_x + \frac{eB}{c}y\right)\left(\frac{\hbar}{i}\partial_x\psi + \frac{eB}{c}y\psi\right)$$

$$= -\hbar^2\partial_x^2\psi + \frac{\hbar}{i}\frac{eB}{c}y\,\partial_x\psi + \frac{\hbar}{i}\frac{eB}{c}y\,\partial_x\psi + \left(\frac{eB}{c}\right)^2 y^2\psi$$

$$= \left(+\hbar^2 k_x^2 + 2\hbar\frac{eB}{c}y\,k_x + \left(\frac{eB}{c}\right)^2 y^2\right)\psi$$

$$\cancel{p_y^2 = \ldots = \hbar^2 \ldots}$$

$$p_z^2\psi = \hbar^2 k_z^2\psi \qquad \Phi$$

$$H\psi \Rightarrow \frac{1}{2\mu}\left[\hbar^2 k_x^2 + \frac{2eB\hbar}{c}y\,k_x + \left(\frac{eB}{c}\right)^2 y^2 + \hbar^2 k_z^2\right]\Phi(y)$$

$$-\frac{\hbar^2}{2\mu}\Phi''(y) = E\,\Phi(y)$$

$$\frac{1}{2\mu}\left[\left(\frac{eB}{c}\right)^2\left(y^2 + \frac{2c}{eB}k_x y + \left(\frac{c}{eB}\right)^2\hbar k_x^2\right) + \hbar^2 k_z^2\right]\Phi$$

$$-\frac{\hbar^2}{2\mu}\Phi'' = E\Phi$$

$$\frac{1}{2\mu}\left[\left(\frac{eB}{c}\right)^2\left[y + \underbrace{\left(\frac{c}{eB}\right)k_x}_{-y_0}\right]^2 + \hbar^2 k_z^2\right]\Phi - \frac{t^2}{2\mu}\Phi'' = E\Phi$$

$$\Phi'' + \frac{2\mu}{\hbar^2}\left[E - \frac{1}{2\mu}\left[\left(\frac{eB}{c}\right)^2 (y-y_0)^2\right] + \hbar^2 k_z^2\right]\Phi = 0$$

$$\Phi'' + \left[\frac{2\mu E}{\hbar^2} - \left[\left(\frac{eB}{\hbar c}\right)^2 (y-y_0)^2\right] + k_z^2\right]\Phi = 0$$

BS K

AKI KUANTUMLANMASI ve AHARANOV-BOHM OLAYI.

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla}+\frac{e}{c}\vec{A}(\vec{r})\right)^2+V(r)\right]\psi(\vec{r})=E\psi(\vec{r})$$

$$\psi(\vec{r})=e^{-i(e/\hbar c)f(\vec{r})}\psi_0(\vec{r})$$ "ayar bozukluğu ortadan kalkar"

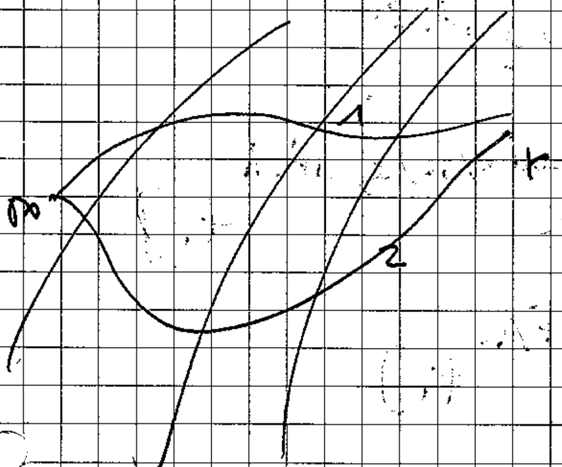
Hamiltoniyen değişmez kalır.

$$\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla}\right)^2\psi_0(r)+V(r)\psi_0(r)=E\psi_0(r)$$

$$\vec{A}\to\vec{A}+\vec{\nabla}f$$

$$f(\vec{r})=\int_{r_0}^{r}d\vec{r}\,'\cdot\vec{A}(\vec{r}\,')$$

$B=0$ ise $\vec{A}=\vec{\nabla}f$



$$\int_1 d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t)-\int_2 d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t)=\oint d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t)$$ (Stokes th.)

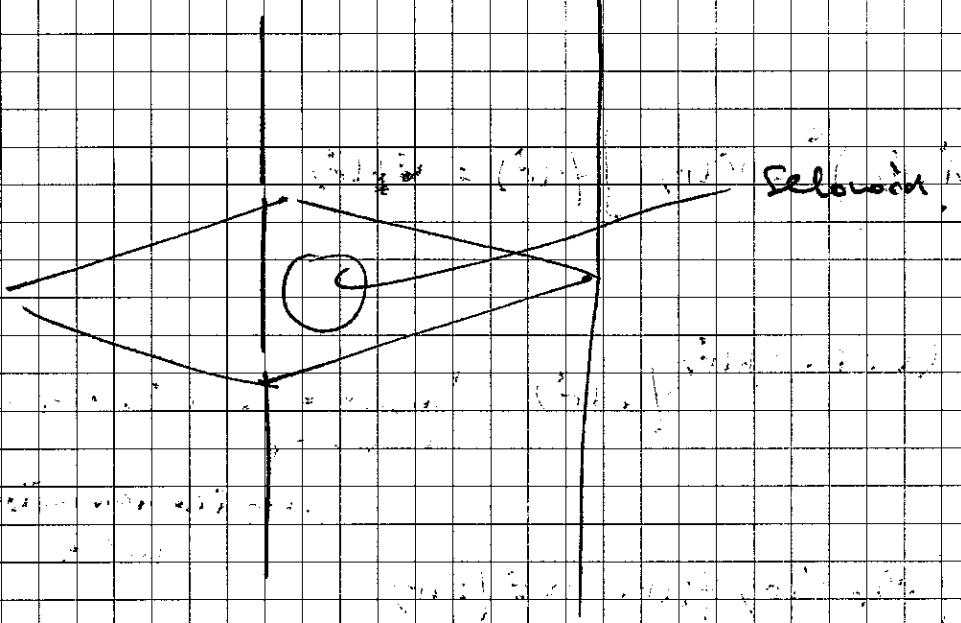
$$=\int_S\left(\vec{\nabla}'\times\vec{A}(\vec{r}\,',t)\right)\cdot d\vec{S}=\int_S\vec{B}\cdot d\vec{S}=\Phi$$

(elekt. dalga fonk. tek değerli olması.)

$$e^{-ie\Phi/\hbar c}=e^{2\pi n i}\Rightarrow\Phi=\frac{2\pi\hbar c}{e}n$$ olmalı.

$n=0,\pm1,\pm2$

Süper iletken olduğunda $\Phi=\frac{2\pi\hbar c}{(2e)}n$.



$\psi = \psi_1 + \psi_2$ megnetik alan yok iken

$$\psi = e^{-(ie/\hbar c)\int_1 d\vec{r}\cdot\vec{A}}\,\psi_1 + e^{-(ie/\hbar c)\int_2 d\vec{r}\cdot\vec{A}}\,\psi_2$$

$$= e^{-(ie/\hbar c)\int_2 d\vec{r}\cdot\vec{A}}\left[\psi_2 + e^{-(ie/\hbar c)\oint d\vec{r}\cdot\vec{A}}\,\psi_1\right]$$

$$= \quad '' \quad \left[\psi_2 + e^{ie\Phi/\hbar c}\,\psi_1\right]$$

Aharonov-Bohm olayı.

AHARANOV-BOHM OLAYI

$$H = \frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2 \qquad H\psi = E\psi$$

$\vec{A} \to \vec{A} + \vec{\nabla} f$ yazmakla $\vec{B} = \vec{\nabla}\times\vec{A}$ aynı kalır.

$$H = \frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A} + \frac{e}{c}\vec{\nabla} f\right)^2$$ 'nin ayarı bozulur.

$\psi(\vec{r},t) \longrightarrow e^{i\Lambda(\vec{r},t)}\psi(\vec{r},t)$ almakla bu ayar bozukluğu ortadan kalkar.

$$\Lambda = -\frac{e}{\hbar c} f$$ Hamiltonien değişmez kalır.

$\vec{B} = \vec{0} \Rightarrow$ yani $\vec{\nabla}\times\vec{A} = \vec{0} \Rightarrow$ yani alan olmayan bir problem ile ilgileniyorsak,

$\vec{A} = \vec{\nabla} f$ olur.

Bu durumda e'nin hareketini iki şekilde seçebiliriz.

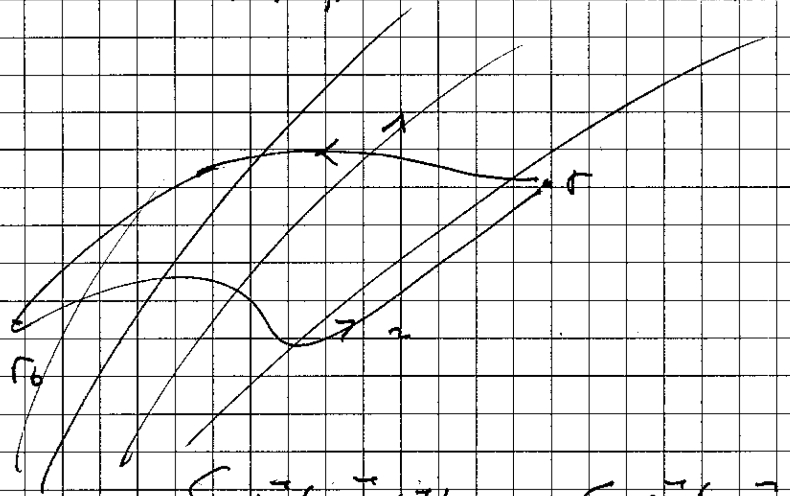
a) $\left[\frac{1}{2\mu}\left[\left(\frac{\hbar}{i}\vec{\nabla}\right)^2\right] + V(\vec{r})\right]\psi = E\psi$

b) $\left[\frac{1}{2\mu}\left[\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2\right] + V(\vec{r})\right]\psi' = E\psi'$

$$\psi' = e^{-ief/\hbar c}\,\psi$$

$$f(\vec{r},t) = \int^{\vec{r}} d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t)$$

Bu integrali $\vec{B}=\vec{0}$ alanını şöylede bir anlamı vardır.

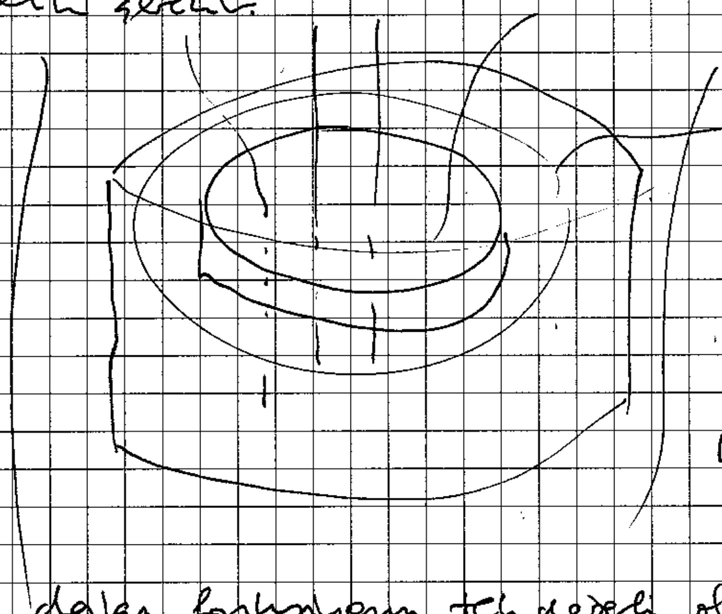


$$\int_1 d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t) + \int_2 d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t) = \oint d\vec{r}\,'\cdot\vec{A}(\vec{r}\,',t)$$

$$= \int_S \left(\vec{\nabla}'\times\vec{A}(\vec{r}\,',t)\right)\cdot d\vec{S} = \int_S \vec{B}\cdot d\vec{S} = \Phi$$

(Stokes yasası)

$\Phi=0 \Rightarrow \psi' = e^{-ief/\hbar c}\,\psi$ deki faz çarpanı çizgi integralindeki yolun seçiminden bağımsız olur. Dalga fonksiyonunun tek değerli olması isteniliyor ise böyle bir bağımsızlık gerekir.



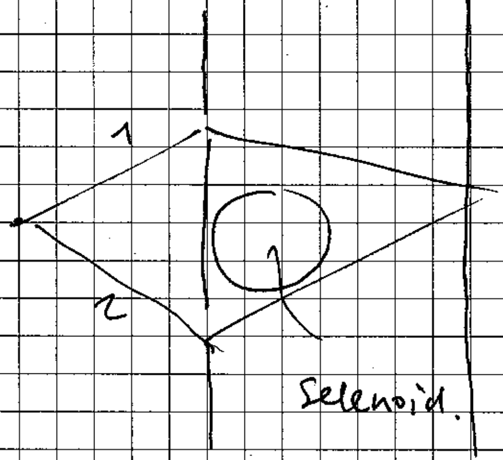
elektron alanın sıfır olduğu yerde hareket ediyor ve bir çevrimi tamamladığında $e^{-ie\Phi/\hbar c}$ faz çarpanı kazanır.

dalga fonksiyonun tek değerli olması için $e^{-ie\Phi/\hbar c} = e^{2\pi n i}$ olmalı

$$\Rightarrow \Phi = \frac{2\pi\hbar c}{e}\,n \qquad n=0,\pm1,\dots$$

BS K

Şöyle bir girişim olayı düşünelim.



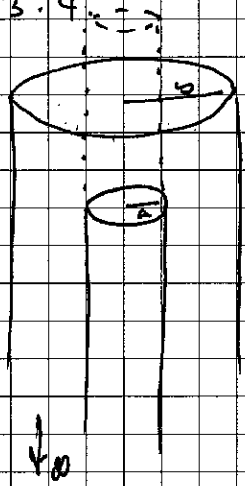
$\psi = \psi_1 + \psi_2$ manyetik alan yok iken

$$\psi = e^{-(ie/\hbar c)\int_1 d\vec{r}\cdot\vec{A}}\,\psi_1 + e^{-(ie/\hbar c)\int_2 d\vec{r}\cdot\vec{A}}\,\psi_2$$

$$= e^{-(ie/\hbar c)\int_2 d\vec{r}\cdot\vec{A}}\left(e^{+ie\Phi/\hbar c}\,\psi_1 + \psi_2 \right)$$ Aharonov-Bohm.

BS K

prb. 13.4



$$\vec{\nabla}^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$-\frac{\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}\right]\psi(\rho,\varphi,z)$$

$$= E\psi(\rho,\varphi,z)$$

$$\psi(\rho,\varphi,z) = u(\rho)\, e^{im\varphi}\, e^{ikz}$$

$$\frac{d^2 u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} + \left(\lambda^2 - \frac{m^2}{\rho^2}\right)u = 0$$

$$u(\rho) = C_1 J_m(\lambda\rho) + C_2 N_m(\lambda\rho)$$

$$u(a) = 0 \qquad u(b) = 0$$

$$C_1 J_m(\lambda a) + C_2 N_m(\lambda a) = 0$$

$$C_1 J_m(\lambda b) + C_2 N_m(\lambda b) = 0$$

$$\frac{J_m(\lambda a)}{J_m(\lambda b)} = \frac{N_m(\lambda a)}{N_m(\lambda b)}$$

BS K

$$i\hbar \frac{d}{dt}\begin{pmatrix} a(t)\\ \\ b(t)\end{pmatrix} = \begin{pmatrix} B_0\, a(t) & B_1 \cos\omega t\, b(t) \\ \\ B_1 \cos\omega t\, a(t) & -B_0\, b(t)\end{pmatrix}\frac{eg\hbar}{4mc}$$

$$\cancel{i\hbar}\,\frac{da}{dt} = \frac{eg\cancel{\hbar}}{4mc}\left[B_0\, a + B_1 \cos\omega t\, b\right]$$

$$\cancel{i\hbar}\,\frac{db}{dt} = \frac{eg\cancel{\hbar}}{4mc}\left[B_1 \cos\omega t\, a - B_0\, b\right]$$

$$egB_0/4mc = \omega_0 \qquad egB_1/4mc = \omega_1$$

$$i\,\frac{da}{dt} = \omega_0\, a + \omega_1 \cos\omega t\, b$$

$$i\,\frac{db}{dt} = \omega_1 \cos\omega t\, a - \omega_1\, b$$

$$\left.\begin{aligned} A(t) &= a(t)\, e^{i\omega_0 t} \\ B(t) &= b(t)\, e^{-i\omega_0 t} \end{aligned}\right\}\ \text{dönüşüm yapalım.}$$

$$i\,\frac{d}{dt}\left(A(t)\, e^{-i\omega_0 t}\right) = \omega_0\, A(t)\, e^{-i\omega_0 t} + \omega_1\, B(t)\, e^{i\omega_0 t} \cos\omega t$$

$$i\,\frac{dA}{dt}\, e^{-i\omega_0 t} + \cancel{\omega_0 A e^{-i\omega_0 t}} = \cancel{\omega_0 A e^{-i\omega_0 t}} + \omega_1\, B\, e^{i\omega_0 t}\cos\omega t$$

$$i\,\frac{dA}{dt} = \omega_1\, B\, e^{2i\omega_0 t}\cos\omega t$$

$$= \frac{\omega_1}{2}\, B\left(\cancel{e^{i(\omega+2\omega_0)t}} + e^{-i(\omega-2\omega_0)t}\right)$$

ihmal.

BS K

$$\frac{dA}{dt} = \frac{\omega_1}{2i} B \, e^{-i(\omega-2\omega_0)t}$$

$$\frac{dB}{dt} = \frac{\omega_1}{2i} A \, e^{i(\omega-2\omega_0)t}$$

$$\frac{d^2A}{dt^2} = \frac{\omega_1}{2i} \underbrace{\frac{dB}{dt}}_{(\omega_1/2i)A} e^{-i(\omega-2\omega_0)t} + \underbrace{\frac{\omega_1}{2i} B \left[-i(\omega-2\omega_0)\right] e^{-i(\omega-2\omega_0)t}}_{dA/dt}$$

$$\frac{d^2A}{dt^2} = \frac{\omega_1}{2i}\frac{\omega_1}{2i} A - i(\omega-\omega_0)\frac{dA}{dt}$$

$$\frac{d^2A}{dt^2} + i(\omega-2\omega_0)\frac{dA}{dt} + \frac{\omega_1^2}{4} A = 0$$

$$\frac{d^2A}{dt^2} - i(2\omega_0-\omega)\frac{dA}{dt} + \frac{\omega_1^2}{4} A = 0$$

$$A(t) = A(0)\, e^{i\lambda t}$$

$$\Rightarrow \quad \lambda_{\mp} = \frac{2\omega_0 - \omega \pm \sqrt{(2\omega_0-\omega)^2+\omega_1^2}}{2}$$

$$A(t) = A_+ e^{i\lambda_+ t} + A_- e^{+i\lambda_- t}$$

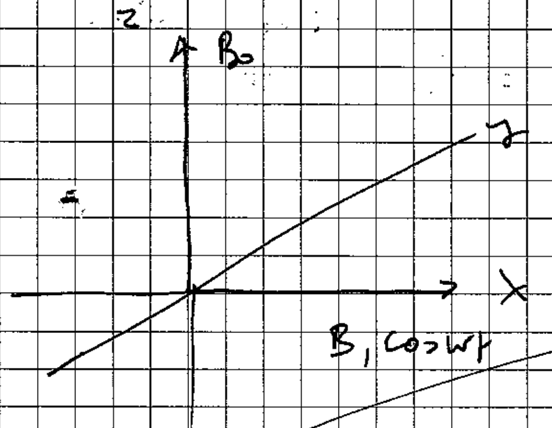
sınır koşullarından sabitler. $a(0)=1$ $b(0)=0$ olsun.

$$\left.\begin{array}{l} A_+ + A_- = 1 \\ \lambda_+ A_+ + \lambda_- A_- = 0 \end{array}\right\} \Rightarrow \quad A_+ = \frac{\lambda_-}{\lambda_- + \lambda_+} \qquad A_- = \frac{\lambda_+}{\lambda_- + \lambda_+}$$

Paramagnetik Rezonans:

Katıda lokalize olmuş tek bir electron için tek bir serbestlik derecesi vardır, o da spindir.

$$i\hbar \frac{d\psi}{\partial t} = \frac{eg\hbar}{4mc} \vec{\sigma}\cdot\vec{B}\,\psi(t)$$



$$\vec{\sigma}\cdot\vec{B} = \sigma_x B_x + \sigma_z B_z$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} B_1 \cos\omega t + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} B_0$$

$$= \begin{pmatrix} B_0 & B_1 \cos\omega t \\ B_1 \cos\omega t & -B_0 \end{pmatrix}$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$ şeklinde

$$i\hbar \frac{d}{dt} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \frac{eg\hbar}{4mc} \begin{pmatrix} B_0 & B_1 \cos\omega t \\ B_1 \cos\omega t & -B_0 \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$t = t$ (>0) anında

$$\langle S_x \rangle = \langle \alpha | S_x | \alpha \rangle$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\omega t} & e^{-i\omega t} \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{-i\omega t} \\ e^{i\omega t} \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$= \frac{\hbar}{4} \begin{pmatrix} e^{i\omega t} & e^{-i\omega t} \end{pmatrix} \begin{pmatrix} e^{i\omega t} \\ e^{-i\omega t} \end{pmatrix} = \frac{\hbar}{4} \left(e^{2i\omega t} + e^{-2i\omega t} \right.$$

$$= \frac{\hbar}{2} \cos 2\omega t$$

$$\langle S_y \rangle = \frac{\hbar}{2} \sin 2\omega t$$

2ω frekansı ile z etrafında presesyon hareketi yapar.

BS K

$$b(t) = B(t)\, e^{i\omega_0 t}$$

$$= -\frac{2}{\omega_1}\, e^{-i(2\omega_0-\omega)t} \left(\lambda_+ A_+ e^{i\lambda_+ t} + \lambda_- A_- e^{i\lambda_- t}\right)$$

$|b(t)|^2$: t anında spinin aşağı yönelme olasılığı.

$$= \frac{\omega_1^2}{(2\omega_0-\omega)^2+\omega_1^2}\; \frac{1-\cos\sqrt{(2\omega_0-\omega)^2+\omega_1^2}\; t}{2}$$

$$\omega = 2\omega_0 \Rightarrow \quad |b(t)|^2 \to \frac{1-\cos\omega_1 t}{2}$$

13. BÖLÜM

PROBLEMLER

13.6.) $H = \frac{1}{2\mu}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2$

$$\frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}\right)\left(\frac{\hbar}{i}\vec{\nabla}\psi + \frac{e}{c}\vec{A}\psi\right) = i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{1}{2\mu}\hbar^2\vec{\nabla}^2\psi + \frac{e\hbar}{2i\mu c}\vec{\nabla}\cdot(\vec{A}\psi) + \frac{e\hbar}{2\mu i c}\vec{A}\cdot\vec{\nabla}\psi + \frac{e^2}{c^2}\vec{A}^2\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi + \frac{e\hbar}{\mu i c}\vec{A}\cdot\vec{\nabla}\psi + \frac{e\hbar}{2i\mu c}(\vec{\nabla}\cdot\vec{A})\psi + \frac{e^2}{c^2}\vec{A}^2\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi - \frac{ie\hbar}{\mu c}\left[\vec{A}\cdot\vec{\nabla} + \frac{1}{2}\vec{\nabla}\cdot\vec{A}\right]\psi + \frac{e^2}{c^2}\vec{A}^2\psi = i\hbar\frac{\partial\psi}{\partial t} \;\Big/\;\psi^*$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}^2\psi^* + \frac{ie\hbar}{\mu c}\left[\vec{A}\cdot\vec{\nabla} + \frac{1}{2}\vec{\nabla}\cdot\vec{A}\right]\psi^* + \frac{e^2}{c^2}\vec{A}^2\psi^* = -i\hbar\frac{\partial\psi^*}{\partial t} \;\Big/\;\psi$$

$$-\frac{\hbar^2}{2\mu}\underbrace{\left(\psi^*\vec{\nabla}^2\psi - \psi\vec{\nabla}^2\psi^*\right)}_{\vec{\nabla}\cdot\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)}$$

$$-\frac{ie\hbar}{\mu c}\Big[\underbrace{\psi^*\vec{A}\cdot\vec{\nabla}\psi + \psi\vec{A}\cdot\vec{\nabla}\psi^*}_{\vec{A}\cdot\vec{\nabla}(\psi\psi^*)} + \underbrace{\frac{1}{2}\psi^*(\vec{\nabla}\cdot\vec{A})\psi + \frac{1}{2}\psi(\vec{\nabla}\cdot\vec{A})\psi^*}_{(\vec{\nabla}\cdot\vec{A})|\psi|^2}\Big] = i\hbar\frac{\partial}{\partial t}(\psi^*\psi)$$

$$-\frac{\hbar^2}{2\mu}\vec{\nabla}\cdot\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{ie\hbar}{\mu c}\left[\vec{A}\cdot\vec{\nabla}(|\psi|^2) + (\vec{\nabla}\cdot\vec{A})|\psi|^2\right] = i\hbar\frac{\partial}{\partial t}|\psi|^2$$

BS K

$$\frac{\partial}{\partial t}|\psi|^2 + \frac{\hbar}{2i\mu}\left\{\vec{\nabla}\cdot\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right)\right\}$$

$$+\frac{e\hbar}{\mu c}\left[\underbrace{\vec{A}\cdot(\vec{\nabla}|\psi|^2) + |\psi|^2(\vec{\nabla}\cdot\vec{A})}_{\vec{\nabla}\cdot\left(\vec{A}|\psi|^2\right)}\right] = 0$$

$$\frac{\partial}{\partial t}|\psi|^2 + \vec{\nabla}\cdot\vec{j} = 0$$

$$\vec{j} = \frac{\hbar}{2i\mu}\left\{\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^* + \frac{2ie}{c}\vec{A}|\psi|^2\right\}$$

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13.7.) $\vec{B} = (0,0,B)$ $\quad \vec{E} = (\mathcal{E},0,0)$

$\vec{A} = (0, xB, 0)$ seçelim. $\qquad \vec{E} = -\vec{\nabla}\phi$

$$H = \frac{1}{2\mu}\left(\vec{p} + \frac{e}{c}\vec{A}\right) + e\phi$$

$$= \frac{1}{2\mu}\left(p_y + \frac{exB}{c}\right)^2 + \frac{p_x^2}{2\mu} + \frac{p_z^2}{2\mu} - e\mathcal{E}x$$

$$= \frac{p_y^2}{2\mu} + \frac{1}{2\mu}\left(\frac{exB}{c}\right)^2 + \frac{1}{\mu}p_y\left(\frac{exB}{c}\right) - e\mathcal{E}x + \frac{p_x^2}{2\mu} + \frac{p_z^2}{2\mu}$$

y ve z yok. $\quad [H, p_y] = [H, p_z] = 0$

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$$\psi = e^{ik_y y + ik_z z}\,\Phi(x)$$

$$H\psi = E\psi \Rightarrow \left[\frac{\hbar^2 k_y^2}{2\mu} + \frac{\hbar k_y}{\mu}\left(\frac{exB}{c}\right) + \frac{1}{2\mu}\left(\frac{exB}{c}\right)^2 - e\mathcal{E}x + \frac{\hbar^2 k_z^2}{2\mu}\right]\Phi$$

$$-\frac{\hbar^2}{2\mu}\Phi'' = E\Phi$$

$$\Phi'' - \frac{2\mu}{\hbar^2}\left\{-E + \frac{\hbar^2 k_z^2}{2\mu} + \frac{\hbar^2 k_y^2}{2\mu} + \frac{\hbar k_y}{\mu}\left(\frac{exB}{c}\right)\right.$$

$$\left. + \frac{1}{2\mu}\left(\frac{exB}{c}\right)^2 - e\mathcal{E}x\right\}\Phi = 0$$

$$\frac{1}{2\mu}\left(\frac{exB}{c}\right)^2 - e\mathcal{E}x + \frac{\hbar k_y}{\mu}\left(\frac{exB}{c}\right)$$

$$= \frac{1}{2\mu}\left(\frac{eB}{c}\right)^2 x^2 + x\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} - e\mathcal{E}\right)$$

$$= \frac{1}{2\mu}\left(\frac{eB}{c}\right)^2\left[x^2 + 2x\mu\left(\frac{c}{eB}\right)^2\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} - e\mathcal{E}\right)\right]$$

$$= \frac{1}{2\mu}\left(\frac{eB}{c}\right)^2\left[x + \mu\left(\frac{c}{eB}\right)^2\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} - e\mathcal{E}\right)\right]^2$$

$$- \frac{1}{2\mu}\cancel{\left(\frac{eB}{c}\right)^2}\cdot\mu^2\left(\frac{c}{eB}\right)^{\cancel{4}\,2}\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} - e\mathcal{E}\right)^2$$

$$= \frac{1}{2\mu}\left(\frac{eB}{c}\right)^2(x + x_0)^2 - \frac{1}{2\cancel{\mu}}\cancel{\mu}\left(\frac{c}{eB}\right)^2\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} - e\mathcal{E}\right)^2$$

$$\Phi'' + \frac{2\mu}{\hbar^2}\left\{E' - \frac{1}{2\mu}\left(\frac{eB}{c}\right)^2(x - x_0)^2\right\}\Phi = 0$$

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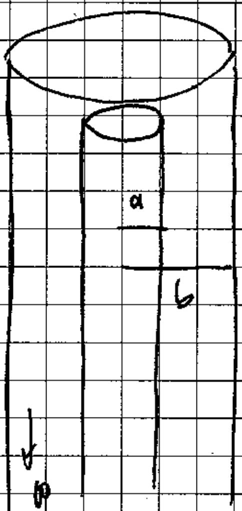
4.

$$E' - \hbar\omega\left(n+\frac{1}{2}\right) = E - \frac{\hbar^2 k_y^2}{2\mu} - \frac{\hbar k_z^2}{2\mu} + \frac{1}{2}\mu\left(\frac{c\phi}{eB}\right)^2\left(\frac{eB}{c}\frac{\hbar k_y}{\mu} e\mathcal{E}\right)^2$$

prb. 13.9.) (4)



$$-\frac{\hbar^2}{2\mu}\nabla^2\psi = E\psi$$

$$-\frac{\hbar^2}{2\mu}\left[\frac{\partial^2\psi}{\partial z^2} + \frac{1}{\rho}\frac{\partial\psi}{\partial\rho} + \frac{\partial^2\psi}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\varphi^2}\right] = E\psi$$

$$\psi = e^{im\varphi} e^{ikz} u(\rho)$$

$$\frac{d^2u}{d\rho^2} + \frac{1}{\rho}\frac{du}{d\rho} + \left(\lambda^2 - \frac{m^2}{\rho^2}\right)u = 0$$

$$\lambda^2 = \frac{2mE}{\hbar^2} - k^2$$

$$R(\rho) = u(\rho) = C_1 J_m(\lambda\rho) + C_2 N_m(\lambda\rho)$$

$$R(a) = 0 \qquad R(b) = 0$$

$$C_1 J_m(\lambda a) + C_2 N_m(\lambda a) = 0$$

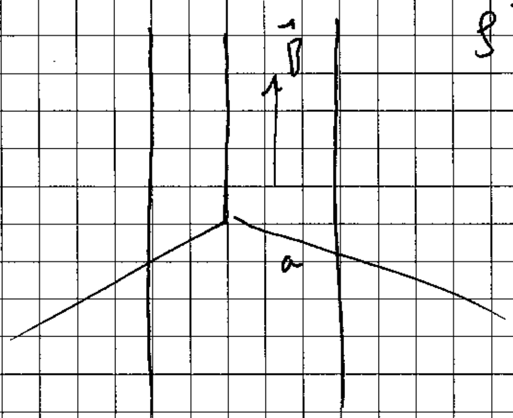
$$C_1 J_m(\lambda b) + C_2 N_m(\lambda b) = 0$$

$$\Rightarrow \quad \frac{J_m(\lambda a)}{J_m(\lambda b)} = \frac{N_m(\lambda a)}{N_m(\lambda b)}$$

BS K

5

prb. 13.8.)



$\rho > a$ $\vec B = \vec 0$ ama $\vec A = \vec 0$ olmak zorunda değil.

$$\vec A(\rho, \varphi, z) = \vec\nabla \Lambda(\rho, \varphi, z)$$

$\Rightarrow$ $\vec\nabla \cdot \vec A = 0$ olsun.

$$\Rightarrow \vec\nabla^2 \Lambda = 0 \Rightarrow \left[\frac{\partial^2}{\partial z^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2}\right]\Lambda = 0$$

Λ sadece φ'ye bağlı

0 seçilebilir.

$$\frac{\partial^2 \Lambda}{\partial \varphi^2} = 0 \Rightarrow \Lambda = C\varphi + \cancel{D}$$

$$\oint d\vec r \cdot \vec A = \Phi = \oint d\vec r \cdot \vec\nabla \Lambda$$

$$d\vec r = d\rho\, \vec e_1 + \rho\, d\varphi\, \vec e_2 + dz\, \vec e_3$$

$$\vec\nabla \psi = \vec e_1 \frac{\partial \psi}{\partial \rho} + \vec e_2 \frac{1}{\rho}\frac{\partial \psi}{\partial \varphi} + \vec e_3 \frac{\partial \psi}{\partial z}$$

$$\oint \rho\, d\varphi \frac{1}{\rho}\frac{d\Lambda}{d\varphi} = \Phi \Rightarrow C\oint d\theta = \Phi \Rightarrow \Phi = C\varphi\Big|_0^{2\pi}$$

$$C = \frac{\Phi}{2\pi} \Rightarrow \Lambda = \frac{\varphi\Phi}{2\pi}.$$

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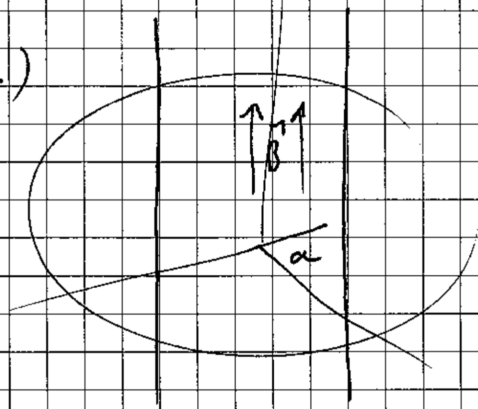
prb. 13.5.)



$\rho > a$ $\vec{B} = \vec{0}$ ama $\vec{A} \neq \vec{0}$ olmalı

zorunda değil!..

$$\vec{A}(\rho\varphi z) = \vec{\nabla}\Lambda(\rho\varphi z)$$

$\vec{\nabla}\cdot\vec{A} = \vec{0}$ seçmek koşulu ile.

$\nabla^2 \Lambda = 0$ verir.

ama Λ'nın sadece φ bağımlılığı var.

$$\Rightarrow \therefore \quad \frac{\partial^2 \Lambda}{\partial \varphi^2} = 0 \Rightarrow \Lambda = B\varphi + \cancel{C}^{\,0}$$

$$\oint d\vec{r}\cdot\vec{A} = \Phi \Rightarrow \Phi = \int d\vec{r}\cdot\vec{\nabla}\Lambda$$

$$d\vec{r} = d\rho\,\hat{e}_1 + \rho\, d\varphi\,\hat{e}_2 + dz\,\hat{e}_3$$

$$\vec{\nabla}\psi = \hat{e}_1 \frac{\partial\psi}{\partial\rho} + \hat{e}_2 \frac{1}{\rho}\frac{\partial\psi}{\partial\varphi} + \hat{e}_3 \frac{\partial\psi}{\partial z}$$

$$\Phi = \int \cancel{\rho}\, d\varphi \frac{1}{\cancel{\rho}}\frac{d\Lambda}{d\varphi} = \int B\, d\varphi = B\varphi\Big|_0^{2\pi} = B\,2\pi \Rightarrow B = \frac{\Phi}{2\pi}$$

$$\Lambda = \frac{\varphi\,\Phi}{2\pi}$$

$$(\mu\,\vec{r}\times\vec{v})_z = L_z = \left[\vec{r}\times\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}\right)\right]_z$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial\varphi} + \frac{e}{c}\frac{\Phi}{2\pi}$$

BS K

BS K