BÖLÜM 14

OPERATÖRLER, MATRİSLER ve SPİN

$$\left.\begin{aligned} \vec{L}^2 Y_{\ell m} &= \hbar^2 \ell(\ell+1) Y_{\ell m} \\ L_z Y_{\ell m} &= \hbar m Y_{\ell m} \end{aligned}\right\} \text{Açısal mom. özdeğer pro.}$$

$$u_n = \frac{1}{\sqrt{n!}} (A^+)^n u_0 \qquad \left\{\begin{aligned} A^+ u_n &= \sqrt{n+1}\, u_{n+1} \\ A u_n &= \sqrt{n}\, u_{n-1} \end{aligned}\right.$$

$$H u_n = \hbar\omega \left(n + \frac{1}{2}\right) u_n$$

ve $\langle u_m | u_n \rangle = \delta_{nm}$ idi.

$$\langle u_m | H u_n \rangle = \langle u_m | H | u_n \rangle = \left(n + \frac{1}{2}\right) \hbar\omega\, \delta_{mn}$$

$$\langle u_m | A^+ u_n \rangle = \langle u_m | A^+ | u_n \rangle = \sqrt{n+1}\, \delta_{m\,n+1}$$

$$\langle u_m | A u_n \rangle = \langle u_m | A | u_n \rangle = \sqrt{n}\, \delta_{m\,n-1}$$

$$H \rightarrow \begin{array}{c|c} & n \rightarrow \quad 0 \quad 1 \quad 2 \quad \cdots \\ \hline m \downarrow & \\ 0 & \\ 1 & \left(\begin{matrix} 1/2 & & & 0 \\ & 3/2 & & \\ & & 5/2 & \\ 0 & & & \ddots \end{matrix}\right) \hbar\omega \\ \vdots & \end{array}$$

2.

$$A^{\dagger} \rightarrow \begin{array}{c|c} & n \rightarrow \\ \hline m \downarrow & \begin{pmatrix} 0 & 0 & & & \\ \sqrt{1} & 0 & & 0 & \\ 0 & \sqrt{2} & & & \\ 0 & & \sqrt{3} & & \\ 0 & & & & \\ \vdots & & 0 & & \end{pmatrix} \end{array} \qquad A \rightarrow \begin{pmatrix} 0 & \sqrt{1} & & 0 \\ & & \sqrt{2} & \\ & & & \sqrt{3} \\ & 0 & & \end{pmatrix}$$

$\langle u_m | F | u_n \rangle$: F herhangi bir op. u_i tamlıkta olmak üzere F'nin u_i tabanına göre matris temsili.

$$(FG)_{ij} = \sum_n (F)_{in} (G)_{nj} \quad ✓$$

$$G u_j = \sum_n c_n u_n \Rightarrow c_n = \langle u_n | G | u_j \rangle$$

$$\langle u_i | FG | u_j \rangle = \langle u_i | F \left(\sum_n c_n u_n \right) \rangle$$

$$= \sum_n c_n \langle u_i | F | u_n \rangle$$

$$✓ \qquad = \sum_n \langle u_i | F | u_n \rangle \langle u_n | G | u_j \rangle$$

✓ aynı $\Leftrightarrow$ $\langle u_i | F | u_n \rangle = F_{in}$

$$\sum_n |u_n\rangle \langle u_n| = I \quad \text{tamlık}$$

BS K

$$\langle u_m | F | u_n \rangle^* = \langle F u_n | u_m \rangle = \langle u_n | F^\dagger | u_m \rangle$$

$$\Rightarrow \left(F^\dagger\right)_{nm} = F^*_{mn}$$

Açısal Mom. Op.'lerinin Matris Temsilleri

Farklı açısal mom. durumları arasında L_z'nin matris elemanına odaklanalım

$$\langle l'm' | L_z | lm \rangle \rightarrow ?$$

$[\vec{L}^2, L_z] = 0$ idi

$$0 = \langle l'm' | [\vec{L}^2, L_z] | lm \rangle$$

$$= \langle l'm' | \vec{L}^2 L_z | lm \rangle - \langle l'm' | L_z \vec{L}^2 | lm \rangle$$

$$= \langle \vec{L}^2 l'm' | L_z | lm \rangle - \langle l'm' | L_z | \vec{L}^2 lm \rangle$$

$$= \{ l'(l'+1) - l(l+1) \} \hbar^2 \langle l'm' | L_z | lm \rangle$$

$$\Rightarrow \left\{ L_z\text{'nin matris elemanı} \right\} \begin{cases} 0 & l \neq l' \\ \neq 0 & l = l' \end{cases}$$

$$\langle l'm' | L_z | lm \rangle = m\hbar \, \delta_{m'm} \, \delta_{l'l}$$

$$\langle lm' | L_z | lm \rangle = m\hbar \, \delta_{mm'}$$

BS K

4.

$$\langle l m' | L_\pm | l m \rangle = \hbar \left[l(l+1) - m(m\pm1) \right]^{1/2} \delta_{m', m\pm1}$$

$l = 1 \Rightarrow m = 0, \pm 1$

$$L_z = \hbar \begin{array}{c c} & \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{array}{c} 1 \\ 0 \\ -1 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

$$L_+ = \hbar \begin{array}{c c} & \begin{array}{ccc} +1 & 0 & -1 \end{array} \\ \begin{array}{c} +1 \\ 0 \\ -1 \end{array} & \begin{bmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad L_- = \begin{array}{c c} & \begin{array}{ccc} 1 & 0 & -1 \end{array} \\ \begin{array}{c} 1 \\ 0 \\ -1 \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \end{array}$$

$$[L_+, L_-] = \hbar^2 \begin{bmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} - \hbar^2 \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \hbar^2 \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \hbar^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = 2\hbar L_z$$

BS K

Durumlar arasındaki genel bağıntılar de matris gösterimi ile yazılabilir.

$$\psi = A\phi$$

olsun.

$$\langle u_i|\psi\rangle = \langle u_i|A\phi\rangle$$

$$= \langle u_i|A\sum_n |u_n\rangle\langle u_n|\phi\rangle$$

$$= \sum_n \langle u_i|A|u_n\rangle\langle u_n|\phi\rangle$$

Eğer $\langle u_n|\phi\rangle$ leri

$$\langle u_n|\phi\rangle \longrightarrow \begin{pmatrix}\langle u_1|\phi\rangle\\ \langle u_2|\phi\rangle\\ \vdots\\ \vdots\end{pmatrix} \equiv \begin{pmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \vdots\end{pmatrix}$$

benzer şekilde

$$\langle u_n|\psi\rangle = \begin{pmatrix}\langle u_1|\psi\rangle\\ \langle u_2|\psi\rangle\\ \vdots\\ \vdots\end{pmatrix} = \begin{pmatrix}\beta_1\\ \beta_2\\ \vdots\\ \vdots\end{pmatrix}$$

şeklinde kolon vektörleri ile gösterirsek

$$\psi = A\phi \longrightarrow \beta_i = \sum_n A_{in}\alpha_n$$

$$\langle\phi|u_n\rangle = \langle u_n|\phi\rangle^* = (\alpha_1^* \ \ \alpha_2^* \ \ldots)$$ satır vektörü olur

$\langle \phi | \psi \rangle$ ne olur?

$$\langle \phi | \psi \rangle = \sum_n \langle \phi | u_n \rangle \langle u_n | \psi \rangle$$

$$= \sum_n \alpha_n^* \beta_n$$

Özdeğer problemi

$$A \phi = a \phi$$

$$\longrightarrow \sum_n A_{in} \alpha_n = a \alpha_i$$

$$\begin{pmatrix} A_{11} - a & A_{12} & A_{13} & \cdots \\ A_{21} & A_{22} - a & A_{23} & \cdots \\ \vdots & \vdots & & \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\det | A_{in} - a \delta_{in} | = 0$$

$$\begin{pmatrix} 8 & 7 & 6 \\ 4 & 14 & 4 \\ 6 & 4 & r \end{pmatrix}$$

$\lambda = 2, 14 \mp 4\sqrt{2}$

Spin Operatörleri ve Matris Temsili:

$$\ell = 1/2$$

$$Y_{\ell\ell} = C\,(\sin\theta)^{\ell}\, e^{i\ell\phi} \qquad Y_{\ell,-m} = (-1)^{m}\, Y_{\ell m}^{*}$$

$$Y_{\ell-\ell} = C\,(-1)^{\ell}\,(\sin\theta)^{\ell}\, e^{-i\ell\phi}$$

$$Y_{\ell,\pm\ell} = C_{\pm}\,(\sin\theta)^{\ell}\, e^{\pm i\ell\phi}$$

$$Y_{1/2\pm1/2} = C_{\pm}\sqrt{\sin\theta}\; e^{\pm i\phi/2}$$

$$L_{-}Y_{\ell\ell} \propto \hbar\, e^{i(\ell-1)\phi}\frac{\partial}{\partial\theta}(\sin\theta)^{\ell}$$

$$L_{-}Y_{1/2-1/2} \propto \frac{\cos\theta}{\sqrt{\sin\theta}}\, e^{-i\phi/2}$$

Ama bu yukarıdaki özdeş değil. $\ell = 1/2$ için güçlükler vardır.

$$[S_x, S_y] = i\hbar S_z$$

$$S_z = \hbar\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$$

$$S_{+} = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad S_{-} = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

BS K

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_x, \sigma_y] = i\sigma_z$$

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$$

$$\{\sigma_i, \sigma_j\} = 0 \quad \begin{cases} \{\sigma_x, \sigma_y\} \\ = \{\sigma_z, \sigma_x\} = \{\sigma_y, \sigma_z\} = 0 \end{cases}$$

$$S_z \begin{pmatrix} u \\ v \end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix} u \\ v \end{pmatrix}$$ — spinor

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \pm\begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \begin{pmatrix} u \\ -v \end{pmatrix} = \pm\begin{pmatrix} u \\ v \end{pmatrix}$$

$(+)$ $$\begin{pmatrix} u \\ -v \end{pmatrix} - \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -2v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v = 0 \text{ olmalı. } \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$(-)$ $$\begin{pmatrix} 2u \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow u = 0 \text{ olmalı. } \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Herhangi bir spinör:

$$\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix} = \alpha_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_- \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\alpha_+|^2 + |\alpha_-|^2 = 1$$

$\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$ durumunda $+1/2$ ölçme desteği.
$-1/2$ " "

S_z'yi diagonal almak şart değildir. $S_x\cos\phi + S_y\sin\phi$ operatörünün özdurumlarını arıyor isek,

$$\left(S_x\cos\phi + S_y\sin\phi\right)\begin{pmatrix} u \\ v \end{pmatrix} = \frac{\hbar}{2}\lambda\begin{pmatrix} u \\ v \end{pmatrix}$$

denklemini çözmeliyiz. Yani,

$$\begin{pmatrix} 0 & \cos\phi - i\sin\phi \\ \cos\phi + i\sin\phi & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \lambda\begin{pmatrix} u \\ v \end{pmatrix}$$

$$\Rightarrow \quad \begin{matrix} v\,e^{-i\phi} = \lambda u \\ u\,e^{i\phi} = \lambda v \end{matrix} \qquad \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \lambda\begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & e^{-i\phi} \\ e^{i\phi} & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

BS K

$\lambda = 1$ özdeğerine karşı gelen öz fonksiyon

$$\begin{pmatrix} -1 & e^{-i\phi} \\ e^{i\phi} & -1 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} -u + v e^{-i\phi} = 0 \\ u e^{i\phi} - v = 0 \end{array}\right\} \; v = u e^{+i\phi}$$

$$\begin{pmatrix} u \\ u e^{+i\phi} \end{pmatrix} = \underbrace{u e^{i\phi/2}}_{\bar{u}} \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}$$

$$\bar{u}^* \left(e^{i\phi/2} \quad e^{-i\phi/2} \right) \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix} \bar{u} = 1$$

$$|\bar{u}|^2 (1+1) = 1 \rightarrow |\bar{u}| = \frac{1}{\sqrt{2}}$$

$$u_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}$$

$$u_- = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ -e^{i\phi/2} \end{pmatrix}$$

$$u_+^* u_- = \frac{1}{2} \left(e^{i\phi/2} \quad e^{-i\phi/2} \right) \begin{pmatrix} e^{-i\phi/2} \\ -e^{i\phi/2} \end{pmatrix} = 0$$

Herhangi bir α durumu verilmiş olsun, $\vec{S}$'nin beklenen değeri hesaplanabilir.

$$\langle \alpha | \vec{S} | \alpha \rangle = \sum_i \sum_j \langle \alpha | i \rangle \langle i | \vec{S} | j \rangle \langle j | \alpha \rangle$$

Ya da eşdeğer olarak

$$\begin{pmatrix} \alpha_+^* & \alpha_-^* \end{pmatrix} \vec{S} \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$\langle S_x \rangle = \begin{pmatrix} \alpha_+^* & \alpha_-^* \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \alpha_+^* & \alpha_-^* \end{pmatrix} \begin{pmatrix} \alpha_- \\ \alpha_+ \end{pmatrix} = \frac{\hbar}{2} \left(\alpha_+^* \alpha_- + \alpha_-^* \alpha_+ \right)$$

$$\langle S_y \rangle = \frac{\hbar}{2} \begin{pmatrix} \alpha_+^* & \alpha_-^* \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix} =$$

$$= \frac{\hbar}{2} \begin{pmatrix} \alpha_+^* & \alpha_-^* \end{pmatrix} \begin{pmatrix} -i\alpha_- \\ +i\alpha_+ \end{pmatrix} = -\frac{i\hbar}{2} \left(\alpha_+^* \alpha_- - \alpha_-^* \alpha_+ \right)$$

$$\langle S_z \rangle = \frac{\hbar}{2} \left(|\alpha_+|^2 + |\alpha_-|^2 \right) = \frac{\hbar}{2}.$$

BS K

Spin 1/2 parçacıkların iç magnetik momenti.

Elektron katı içerisinde yapılı ile, elektronun serbestlik derecesi sadece spin olabilir.

$\vec{L}$ açısal mom.'lu bir ele benzer yörüngede hareket eden bir elektron magnetik momenti

$$\vec{M} = -\frac{e}{2mc}\vec{L}$$

olan bir akım halkası oluşturur. Analoji ile,

$$\vec{M} = -\frac{eg}{2mc}\vec{S} \qquad g = 2\left(1+\frac{\alpha}{2\pi}+\dots\right) = 2.0023192$$

$$H = -\vec{M}\cdot\vec{B} = \frac{eg}{2mc}\vec{S}\cdot\vec{B} = \frac{eg\hbar}{4mc}\vec{\sigma}\cdot\vec{B}$$

$H\psi = i\hbar\frac{\partial\psi}{\partial t}$ denklemini ~~[illegible]~~ çözmek istiyorum.

$$\psi = \begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix} = \frac{eg\hbar}{4mc}\vec{\sigma}\cdot\vec{B}\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix}$$

$\vec{B} = B_0\hat{z}$ olsun.

$$\Psi(t) = e^{-i\omega t}\begin{pmatrix}\alpha_+(0)\\ \alpha_-(0)\end{pmatrix} \qquad \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$i\frac{\partial}{\partial t}\left[e^{-i\omega t}\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix}\right] = \frac{egB}{4mc}\sigma_z\, e^{-i\omega t}\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix}$$

$$\omega\, \cancel{e^{-i\omega t}}\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix} = \frac{egB}{4mc}\, \cancel{e^{-i\omega t}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix}$$

$$\omega\begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix} = \frac{egB}{4mc}\begin{pmatrix}\alpha_+\\ -\alpha_-\end{pmatrix}$$

$$\omega = \frac{egB}{4mc} \Rightarrow \begin{pmatrix}\alpha_+\\ \alpha_-\end{pmatrix} = \begin{pmatrix}\alpha_+\\ -\alpha_-\end{pmatrix} \Rightarrow \begin{pmatrix}0\\ -2\alpha_-\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\alpha_- = 0$$

$$\Psi(t) = e^{-i\omega t}\begin{pmatrix}\alpha_+\\ 0\end{pmatrix} = e^{-i\omega t}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\omega = -\frac{egB}{4mc} \Rightarrow \Psi(t) = e^{-i\omega t}\begin{pmatrix}0\\ 1\end{pmatrix}$$

Eğer $t=0$ anında ilk durum $\Psi(0) = \begin{pmatrix}a\\ b\end{pmatrix}$ olsa idi

$$\Psi(t) = \begin{pmatrix}a\, e^{-i\omega t}\\ b\, e^{i\omega t}\end{pmatrix} \text{ olurdu.}$$

BS K

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$$i\hbar\frac{\partial}{\partial t}\Psi = \frac{eg\hbar}{4mc}\vec{\sigma}\cdot\vec{B}\,\Psi = \frac{eg\hbar B}{4mc}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\psi$$

$$\psi(t) = \begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix} = \frac{eg\hbar B}{4mc}\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix}$$

$$i\cancel{\hbar}\frac{\partial}{\partial t}\alpha_+(t) = \overbrace{\frac{egB}{4mc}}^{\omega}\cancel{\hbar}\,\alpha_+(t) \Rightarrow \alpha_+(t) = \alpha_+(0)\,e^{-i\omega t}$$

$$i\frac{\partial}{\partial t}\alpha_-(t) = \frac{-egB}{4mc}\alpha_-(t) \Rightarrow \alpha_-(t) = \alpha_-(0)\,e^{+i\omega t}$$

$t=0$ da spin $+\hbar/2$ özdeğerli S_x'in bir özdurumunda olsun. Yani x-doğrultusunda yönelmiş $\psi(0) = \begin{pmatrix}a\\ b\end{pmatrix}$

$$\cancel{\frac{\hbar}{2}}\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{\sigma_x}\begin{pmatrix}a\\ b\end{pmatrix} = \cancel{\frac{\hbar}{2}}\begin{pmatrix}a\\ b\end{pmatrix}$$

$$\begin{pmatrix}b\\ a\end{pmatrix} = \begin{pmatrix}a\\ b\end{pmatrix} \Rightarrow a=b \Rightarrow \psi = \begin{pmatrix}a\\ a\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

BS K

14. BÖLÜM

PROBLEMLER

Prb. 14.1.) $u_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$ taban durumu,

$$A^+ = \begin{pmatrix} 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad A^+ u_0 = u_1$$

$$u_1 = \begin{pmatrix} 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \cdots \\ \vdots & & & \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix} = \sqrt{1} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$$A^+ u_1 = \sqrt{2}\, u_2 \Rightarrow u_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

$$\langle u_1 | u_1 \rangle = 1 \qquad \langle u_1 | u_2 \rangle = 0 \Rightarrow (u_n | u_m) = \delta_{nm}$$

Prb. 14.2.)

$$\Psi = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} \qquad H = \hbar\omega \begin{pmatrix} 1/2 & & & 0 \\ & 3/2 & & \\ & & 5/2 & \\ 0 & & & \ddots \end{pmatrix}$$

$$\langle H \rangle = \langle \Psi | H | \Psi \rangle = \frac{1}{6} \begin{pmatrix} 1 & 2 & 1 & 0 & 0 & \cdots \end{pmatrix} \hbar\omega \begin{pmatrix} 1/2 & & & 0 \\ & 3/2 & & \\ & & 5/2 & \\ 0 & & & \ddots \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$$= \frac{\hbar\omega}{6} \begin{pmatrix} 1 & 2 & 1 & 0 & 0 & \cdots \end{pmatrix} \begin{pmatrix} 1/2 \\ 3 \\ 5/2 \\ 0 \\ \vdots \end{pmatrix} = \frac{\hbar\omega}{6} \left(\frac{1}{2} + 6 + \frac{5}{2} + 0 \cdots \right) = \frac{9}{6} \hbar\omega = \frac{3}{2} \hbar\omega.$$

2

(4) $A^{\dagger} = \sqrt{\frac{m\omega}{2}}\, x + i \frac{p}{\sqrt{2m\omega}}$ $\quad A = \sqrt{\frac{m\omega}{2}}\, x - i \frac{p}{\sqrt{2m\omega}}$

$$x = \frac{1}{\sqrt{2m\omega}} (A + A^{\dagger})$$

$$\langle x \rangle = \langle \psi | x | \psi \rangle = \frac{1}{6} (1\ 2\ 1\ 0\ 0 \ldots) \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \\ \vdots & \vdots & \sqrt{3} & & \ddots \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$$

$$= \frac{1}{6} (1\ 2\ 1\ 0\ 0\ 0 \ldots) \begin{pmatrix} 2 \\ \sqrt{1}+\sqrt{2} \\ 2\sqrt{2} \\ \vdots \end{pmatrix} = \frac{1}{6} \left(2 + 2 + 2\sqrt{2} \right)$$

$$\langle x^2 \rangle = \langle \psi | x x | \psi \rangle$$

$$x|\psi\rangle = \begin{pmatrix} 2 \\ 1+\sqrt{2} \\ 2\sqrt{2} \\ \vdots \end{pmatrix} \Rightarrow \langle \psi | x^{*} = \left(x|\psi\rangle \right)^{\dagger}$$

$$= \left(2 + 1+\sqrt{2}\ \ 2\sqrt{2} \ldots \right)$$

$$\langle \psi | x | \psi \rangle = \frac{15 + 2\sqrt{2}}{6}$$

$$(\Delta x)^2 = \langle x \rangle^2 - \langle x^2 \rangle$$

BS K

14.3.

$$X = \frac{1}{\sqrt{2m\omega}} \left(A + A^{+} \right)$$

$$X^2 = \frac{1}{2m\omega} \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 \\ \sqrt{1} & 0 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} \\ \vdots & \vdots & \sqrt{3} & \ddots \end{pmatrix}$$

$$X^4 = \frac{1}{4m^2\omega^2} \begin{pmatrix} \end{pmatrix} \begin{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \boxed{4 \times 4} & \\ & \end{pmatrix}$$

pr^4. 14. 4.) $\ell = 3/2$ için L_x, L_y, L_z'nin matris gösterimi

$2\ell + 1 = 2 \cdot \frac{3}{2} + 1 = 4$ matris 4×4'lük olmalı

$$\langle \ell m' | L_z | \ell m \rangle = m\hbar \, \delta_{mm'}$$

$$\langle \ell m' | L_{\pm} | \ell m \rangle = \hbar \sqrt{\ell(\ell+1) - m(m \pm 1)} \, \delta_{m' \, m \pm 1}$$

BS K

$$L_z = \hbar \begin{array}{c|c} m' \downarrow \; m \rightarrow & \begin{matrix} 3/2 & 1/2 & -1/2 & -3/2 \end{matrix} \\ \hline \begin{matrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{matrix} & \begin{bmatrix} 3/2 & & 0 & \\ & 1/2 & & \\ & & -1/2 & \\ & 0 & & -3/2 \end{bmatrix} \end{array}$$

$$\left(\frac{3}{2}\,\frac{5}{2} - m(m+1)\right)^{1/2} \delta_{m', m+1}$$

$$L_+ = \hbar \begin{array}{c|c} m' \downarrow \; m \rightarrow & \begin{matrix} 3/2 & 1/2 & -1/2 & -3/2 \end{matrix} \\ \hline \begin{matrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{matrix} & \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & \sqrt{4} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\left[\frac{15}{4} - m(m-1)\right]^{1/2} \delta_{m', m-1}$$

$$L_- = \hbar \begin{array}{c|c} m' \downarrow \; m \rightarrow & \\ \hline \begin{matrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{4} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix} \end{array}$$

$$L_x = \frac{1}{2}\left(L_+ + L_-\right) \qquad iL_y = \frac{1}{2}\left(L_+ - L_-\right)$$

$$= \frac{\hbar}{2} \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & \sqrt{4} & 0 \\ 0 & \sqrt{4} & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix} \qquad = \frac{\hbar}{2} \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ -\sqrt{3} & 0 & \sqrt{4} & 0 \\ 0 & -\sqrt{4} & 0 & \sqrt{3} \\ 0 & 0 & -\sqrt{3} & 0 \end{bmatrix}$$

$$[L_x, L_y] = i\hbar L_z$$

BS K

prb. 14.5.) $H = \frac{1}{2I_1} L_x^2 + \frac{1}{2I_2} L_y^2 + \frac{1}{2I_3} L_z^2$

a) $\ell = 1$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$L_x = \hbar \begin{pmatrix} 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 \end{pmatrix} \qquad L_y = \frac{\hbar}{i} \begin{pmatrix} 0 & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 0 \end{pmatrix}$$

$$H = \frac{\hbar^2}{2} \begin{pmatrix} \frac{1}{2I_1} - \frac{1}{2I_2} + \frac{1}{I_3} & 0 & \frac{1}{2I_1} + \frac{1}{2I_2} \\ 0 & \frac{1}{I_1} - \frac{1}{I_2} & 0 \\ \frac{1}{2I_1} + \frac{1}{2I_2} & 0 & \frac{1}{2I_1} - \frac{1}{2I_2} + \frac{1}{I_3} \end{pmatrix}$$

$$= \begin{pmatrix} a & 0 & c \\ 0 & b & 0 \\ c & 0 & a \end{pmatrix} \qquad (a-\lambda)(b-\lambda)(a-\lambda) + c(-c)(b-\lambda)$$

$H\psi = \lambda\psi \Rightarrow \det(H - \lambda I) = 0$

$$\begin{vmatrix} a-\lambda & 0 & c \\ 0 & b-\lambda & 0 \\ c & 0 & a-\lambda \end{vmatrix} = (b-\lambda)\left[(a-\lambda)^2 - c^2\right] = 0$$

$\lambda_1 = b \;,\; (a-\lambda) = \pm c$

$\Rightarrow \lambda_{2,3} = a \pm c$

BS K

6

$$\lambda_1 = \frac{\hbar^2}{2}\left(\frac{1}{I_1} - \frac{1}{I_2}\right)$$

$$\lambda_2 = \frac{\hbar^2}{2}\left(\frac{1}{I_1} + \frac{1}{I_3}\right) \qquad \lambda_3 = \frac{\hbar^2}{2}\left(\frac{1}{I_3} - \frac{1}{I_2}\right)$$

7

prob (4/10)

$$\ell = 1 \qquad u = \frac{1}{\sqrt{26}}\begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$$

L_x 'in ölçümünün 0 değerini verme olasılığı ?

$$L_z = \hbar \begin{bmatrix} 1 & & 0 \\ & 0 & \\ 0 & & -1 \end{bmatrix}$$

$$L_x = \hbar \begin{bmatrix} 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$L_x \psi = \lambda \psi \rightarrow \det(L_x - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & \hbar/\sqrt{2} & 0 \\ \hbar/\sqrt{2} & -\lambda & \hbar/\sqrt{2} \\ 0 & \hbar/\sqrt{2} & -\lambda \end{vmatrix} = 0 \qquad \cancel{-\lambda\left(\lambda^2 - \frac{\hbar}{\sqrt{2}}\right) - \hbar} \qquad \cancel{-\lambda(\lambda^2 - 0) - \frac{\hbar}{2}}$$

BS K

$$-\lambda\left(\lambda^2-\frac{t_1^2}{2}\right)-\frac{t_1}{\sqrt{2}}\left(-\lambda\frac{t_1}{\sqrt{2}}\right)=0$$

$$-\lambda\left(\lambda^2-\frac{t_1^2}{2}\right)+\lambda\frac{t_1^2}{2}=0$$

$$-\lambda\left(\lambda^2-\frac{t_1^2}{2}-\frac{t_1^2}{2}\right)=0 \qquad \lambda_1=0$$

$$\lambda^2-t_1^2=0 \Rightarrow \lambda_{2,3}=\pm t_1$$

λ_1 ↘

$$t_1\begin{pmatrix}0 & 1/\sqrt{2} & 0\\ 1/\sqrt{2} & 0 & 1/\sqrt{2}\\ 0 & 1/\sqrt{2} & 0\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}\Rightarrow t_1\begin{pmatrix}b/\sqrt{2}\\ a/\sqrt{2}+c/\sqrt{2}\\ b/\sqrt{2}\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\frac{b}{\sqrt{2}}=0 \qquad \frac{a}{\sqrt{2}}+\frac{c}{\sqrt{2}}=0 \qquad \frac{b}{\sqrt{2}}=0 \Rightarrow \left\{ b=0,\ a=-c \right.$$

$$\lambda_1=0 \Rightarrow u_1=N\begin{pmatrix}a\\ 0\\ -a\end{pmatrix} \qquad u_1^* u_1=1\Rightarrow\begin{pmatrix}a & 0 & -a\end{pmatrix}\begin{pmatrix}a\\ 0\\ -a\end{pmatrix}$$

$$\Rightarrow a^2+a^2=1 \Rightarrow a=\frac{1}{\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\\ -1\end{pmatrix}$$

$$\lambda_2=t_1 \Rightarrow \qquad u_2=\begin{pmatrix}1/2\\ 1/\sqrt{2}\\ 1/2\end{pmatrix} \qquad u=Au_1+Bu_2+Cu_3.$$

$|A|^2$: Supr. člane clasični

$$\lambda_3=-t_1 \Rightarrow \qquad u_3=\begin{pmatrix}1/2\\ -1/\sqrt{2}\\ 1/2\end{pmatrix}$$

8

$$\frac{1}{\sqrt{26}}\begin{pmatrix}1\\4\\-3\end{pmatrix} = A\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix} + B\begin{pmatrix}1/2\\1/\sqrt{2}\\1/2\end{pmatrix} + C\begin{pmatrix}1/2\\-1/\sqrt{2}\\1/2\end{pmatrix}$$

$$\frac{1}{\sqrt{26}} = \frac{A}{\sqrt{2}} + \frac{B+C}{2} \quad (1)$$

$$\frac{4}{\sqrt{26}} = \frac{B-C}{\sqrt{2}}$$

$$-\frac{3}{\sqrt{26}} = -\frac{A}{\sqrt{2}} + \frac{B+C}{2} \quad (2)$$

$(1)-(2)$

$$\frac{1}{\sqrt{26}} + \frac{3}{\sqrt{26}} = \frac{2A}{\sqrt{2}}$$

$$\Rightarrow \frac{\cancel{4}^{\,2}}{\sqrt{26}} = \frac{\cancel{2}A}{\sqrt{2}} \Rightarrow A = \frac{2}{\sqrt{13}} \Rightarrow A^2 = \frac{2}{13} = 0.3$$

8d

Prb. 14.11)

$\ell = 1$ $L_x L_y + L_y L_x$ 'in özfonk.'ları ve

özdeğerleri nedir?

$$L_x L_y + L_y L_x = \frac{\hbar^2}{i}\begin{pmatrix}0 & 0 & 1\\0 & 0 & 0\\-1 & 0 & 0\end{pmatrix}$$

$$\begin{vmatrix}-\lambda & 0 & \hbar^2/i\\0 & -\lambda & 0\\-\hbar^2/i & 0 & -\lambda\end{vmatrix} = 0 \qquad \lambda_1 = 0 \qquad \lambda_{2,3} = \pm\hbar^2$$

BS K

(14.9.) $S = 1/2$.

$S_x + S_y$ 'nin özvektörleri:

$$S_x = \frac{\hbar}{2}\sigma_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\sigma_y \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_x + S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}$$

$$(S_x + S_y)\Psi = \lambda \Psi \Rightarrow \det\left[\left(S_x + S_y\right) + \lambda I\right] = 0$$

$$\frac{\hbar}{2}\begin{vmatrix} 0-\lambda & 1-i \\ 1+i & 0-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \underbrace{(1-i)(1+i)}_{2} = 0$$

$$\lambda_{1,2} = \pm\sqrt{2}$$

$\lambda_1 = \sqrt{2}$

$$\frac{\hbar}{2}\begin{pmatrix} -\sqrt{2} & 1-i \\ 1+i & -\sqrt{2} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-\cancel{\frac{\hbar}{2}}\sqrt{2}a + \cancel{\frac{\hbar}{2}}(1-i)b = 0 \Rightarrow \sqrt{2}a = (1-i)b$$

$$\cancel{\frac{\hbar}{2}}(1+i)a - \cancel{\frac{\hbar}{2}}\sqrt{2}b = 0 \Rightarrow \sqrt{2}b = (1+i)a$$

$=$

$$u = \begin{pmatrix} a \\ b \end{pmatrix} = a\begin{pmatrix} 1 \\ \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

$$u^{*}u = 1 \Rightarrow a^2\left(1 \;\; \frac{1-i}{\sqrt{2}}\right)\begin{pmatrix} 1 \\ \frac{1+i}{\sqrt{2}} \end{pmatrix} = a^2(1+1) = 1 \Rightarrow a = \frac{1}{\sqrt{2}}$$

BS K

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \frac{1+i}{\sqrt{2}} \end{pmatrix} = A\begin{pmatrix} 1 \\ 0 \end{pmatrix} + B\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$|A|^2$ S_z'nin $\uparrow/\downarrow$ ölçme olasılığı

$$\frac{1}{\sqrt{2}} = A$$

14.10.)

$$\frac{dA}{dt} = \frac{i}{\hbar}[H, A]$$

$$H = \frac{eg}{2mc}\vec{S}\cdot\vec{B} \qquad \vec{S} = \vec{S}(t)$$

$$[S_x(t), S_y(t)] = i\hbar S_z(t)$$

$$H = \frac{eg}{2mc}(S_x B_x + S_y B_y + S_z B_z)$$

$$\frac{dS_x}{dt} = \frac{i}{\hbar}[H, S_x]$$

$$= \frac{i}{\hbar}\frac{eg}{2mc}\left\{\cancel{[S_x B_x, S_x]} + [S_y, S_x]B_y + [S_z, S_x]B_z\right\}$$

$$= \frac{i}{\cancel{\hbar}}\frac{eg}{2mc}\left\{-i\cancel{\hbar} S_z B_y + i\cancel{\hbar} S_y B_z\right\}$$

$$= \frac{eg}{\cancel{\hbar}\,2mc}(S_z B_y - S_y B_z)$$

$$= \frac{eg}{2mc}(B_y S_z - B_z S_y)$$

$$\frac{dS_z}{dt} = \frac{eg}{2mc}(B_x S_y - B_y S_x)$$

$$\frac{dS_y}{dt} = \frac{eg}{2mc}(B_z S_x - B_x S_z)$$

BS K

$$\vec{B} = (0, 0, B)$$

$$\Rightarrow \quad \frac{dS_x}{dt} = -\frac{egB}{2mc} S_y$$

$$\frac{dS_y}{dt} = \frac{egB}{2mc} S_x$$

$S_x + iS_y$ 'yi oluşturalım

$$\frac{dS_z}{dt} = 0$$

$$\frac{d}{dt} S_+ = \frac{egB}{2mc}\left(-S_y + iS_x\right)$$

$$= i\frac{egB}{2mc}\left(S_x + iS_y\right) = i\frac{egB}{2mc} S_+$$

$$\frac{dS_+}{dt} = i\omega S_+$$

$\vec{S}(0)$ verilmiş olsun. $S_+(0)$ verilmiş demektir.

$$S_+(t) = S_+(0)\, e^{i\omega t}$$

$$S_x(t) + iS_y(t) = \left(S_x(0) + iS_y(0)\right) \underbrace{e^{i\omega t}}_{\cos\omega t + i\sin\omega t}$$

$$S_x(t) = S_x(0)\cos\omega t - S_y(0)\sin\omega t$$

$$S_y(t) = S_x(0)\sin\omega t + S_y(0)\cos\omega t$$

$$S_z(t) = Sbt. = S_z(0)$$

BS K

14.11. $t=0$ da S_x'in $+\hbar/2$ özdurumunda,

$\vec{B}=(0,0,B)$ alanına konulmuş ve T zaman ile eşi[illegible] [illegible] bırakılmıştır.

$\to \vec{B}=(0,B,0)$ olacak şekilde döndürülmüş ve sonraki T zaman aralığında S_x'i ölçümü yapılıyor. $\hbar/2$ bulma olasılığı nedir?

$t=2T$.

S_x'in beklenen değerinin $\hbar/2$ olma olasılığı?

$$\langle S_x\rangle=\hbar/2\cos 2\omega t$$

$t=0$ $\qquad$ $t=t$ $\qquad$ $t=T$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega t}\\e^{i\omega t}\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}e^{-i\omega T}\\e^{i\omega T}\end{pmatrix}$$

$$\langle S_x\rangle=\frac{\hbar}{2}\cos 2\omega T$$

$$\cancel{i\hbar}\frac{\partial\psi}{\partial t}=\frac{eg\hbar}{4mc}\vec{\sigma}\cdot\vec{B}\,\psi$$

$$=\frac{eg\cancel{\hbar}B}{4mc}\sigma_y\,\psi$$

$$\cancel{i}\frac{d}{dt}\begin{pmatrix}\alpha_+\\\alpha_-\end{pmatrix}=\frac{egB}{4mc}\begin{pmatrix}0&\cancel{-i}\\\cancel{i}&0\end{pmatrix}\begin{pmatrix}\alpha_+\\\alpha_-\end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix}\alpha_+\\\alpha_-\end{pmatrix}=\frac{\omega}{2}\begin{pmatrix}-\alpha_-\\\alpha_+\end{pmatrix}$$

BS K

$$\frac{d\alpha_+}{dt} = -\frac{\omega}{2}\alpha_-$$

$$\frac{d\alpha_-}{dt} = \frac{\omega}{2}\alpha_+$$

$$\frac{d}{dt}\big(\underbrace{\alpha_+ + i\alpha_-}_{\beta}\big) = \frac{\omega}{2}\big(-\alpha_- + i\alpha_+\big)$$

$$= i\frac{\omega}{2}\big(\underbrace{\alpha_+ + i\alpha_-}_{\beta}\big)$$

$$\frac{d\beta}{dt} = i\frac{\omega}{2}\beta \qquad \beta(t) = C\,e^{i\omega t/2}$$

$$\beta(T) = C\,e^{i\omega T/2} \Rightarrow C = \underbrace{\beta(T)}_{\alpha_+(T)+i\alpha_-(T)}\,e^{-i\omega T/2}$$

$$\alpha_+(t) + i\alpha_-(t) = \big[\alpha_+(T) + i\alpha_-(T)\big]\,e^{-i\omega T/2}\,e^{i\omega t/2}$$

$$= [\quad]\,e^{i\frac{\omega}{2}(t-T)}$$

$$\alpha_+(t) = \alpha_+(T)\cos\frac{\omega}{2}(t-T) - \alpha_-(T)\sin\frac{\omega}{2}(t-T)$$

$$\alpha_-(t) = \alpha_-(T)\cos\frac{\omega}{2}(t-T) + \alpha_+(T)\sin\frac{\omega}{2}(t-T)$$

$$\begin{pmatrix}\alpha_+(t)\\ \alpha_-(t)\end{pmatrix} = A\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} + B\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

BS K

BS K