11. BÖLÜM

FERMİ SİSTEMLERİ

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z)$$

$$\frac{1}{v} = \frac{1}{\lambda^3} f_{3/2}(z)$$

$$f_{3/2}(z) = \frac{4}{\sqrt{\pi}} \int_0^\infty dx \, \frac{x^2}{z^{-1} e^{x^2} + 1}$$

küçük z'ler için; $f_{3/2}(z) = z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} - \frac{z^4}{4^{3/2}} + \dots$

büyük z'ler için: Sommerfeld yönt: $z = e^\nu$ de.

$$\ln z = \nu = \frac{\mu}{kT}$$

$$f_{3/2} = \frac{4}{\sqrt{\pi}} \int dx \, \frac{x^2}{e^{x^2-\nu}+1} \qquad x^2 = y \qquad x = \sqrt{y}$$

$$= \frac{2}{\sqrt{\pi}} \int_0^\infty dy \, \frac{\sqrt{y}}{e^{y-\nu}+1}$$

$\sqrt{y}$ yi Taylor [illegible]

$$= \frac{2}{\sqrt{\pi}} \left\{ \cancel{\frac{1}{e^{y-\nu}+1} \cdot \frac{2}{3} y^{3/2} \Big|_0^\infty} - \int_0^\infty \frac{2}{3} \frac{dy \, y^{3/2} (-e^{y-\nu})}{(e^{y-\nu}+1)^2} \right\}$$

$$= \frac{4}{3\sqrt{\pi}} \int_0^\infty dy \, \frac{y^{3/2} e^{y-\nu}}{(e^{y-\nu}+1)^2}$$

$$= \frac{4}{3\sqrt{\pi}} \int_{-\nu}^\infty dt \, \frac{(t+\nu)^{3/2} e^t}{(e^t+1)^2}$$

BS K

$$= \frac{4}{3\sqrt{\pi}} \int_{-\nu}^{\infty} dt \, \frac{e^t}{(e^t+1)^2} \left[\nu^{3/2} + \frac{3}{2}\nu^{1/2} t + \frac{3}{8} \nu^{-1/2} t^2 + \dots \right]$$

$$\int_{-\nu}^{\infty} = \int_{-\infty}^{+\infty} - \int_{-\infty}^{-\nu} \sim e^{-\nu}$$ mertebesinde

$$f_{3/2}(z) = \frac{4}{3\sqrt{\pi}} \int_{-\infty}^{+\infty} dt \, \frac{e^t}{(e^t+1)^2} \left(\nu^{3/2} + \frac{3}{2}\nu^{1/2} t + \frac{3}{8}\nu^{-1/2} t^2 + \dots \right) + O(e^{-\nu})$$

$$= \frac{4}{3\sqrt{\pi}} \left(I_0 \nu^{3/2} + \frac{3}{2} I_1 \nu^{1/2} + \frac{3}{8} I_2 \nu^{-1/2} + \dots \right) + O(e^{-\nu})$$

$$I_n = \int_{-\infty}^{+\infty} dt \, \frac{t^n e^t}{(e^t+1)^2} = \begin{cases} 0 & n \text{ tek} \\ f. & \text{çift} \end{cases}$$

$$I_0 = -2 \int_0^{\infty} dt \, \frac{d}{dt} \frac{1}{(e^t+1)} = 1$$

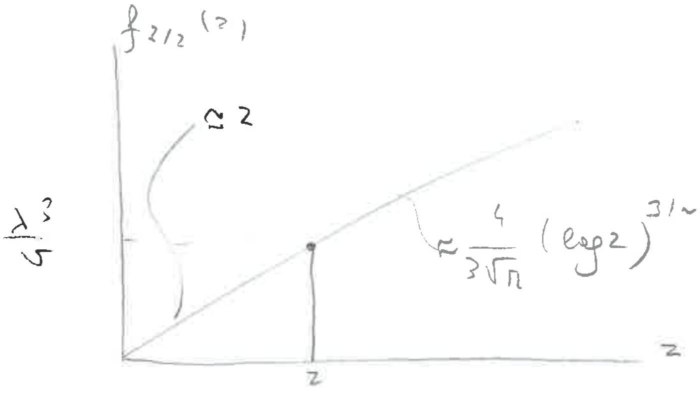
$$I_n = -2 \left[\frac{\partial}{\partial \lambda} \int_0^{\infty} dt \, \frac{t^{n-1}}{e^{\lambda t}+1} \right]_{\lambda=1} = 2n \int_0^{\infty} du \, \frac{u^{n-1}}{e^u+1}$$

$$= (n-1)! \, (2n) \, (1 - 2^{1-n}) \, \zeta(n)$$ Riemann Zeta fonk.

$$\zeta(2) = \frac{\pi^2}{6} \qquad \zeta(4) = \frac{\pi^4}{90} \qquad \zeta(6) = \frac{\pi^6}{945}$$

$$f_{3/2}(z) = \frac{4}{3\sqrt{\pi}} \left[(\log z)^{3/2} + \frac{\pi^2}{8} (\log z)^{-1/2} + \dots \right] + O(z^{-1})$$

BS K



Yüksek sıcaklık & Düşük yoğunluk ($\lambda^3/v \ll 1$)

$$\frac{\lambda^3}{v} = z - \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \cdots \qquad \ll 1$$

İlk yaklaşıklıkta: $z = \frac{\lambda^3}{v} + \frac{1}{2^{3/2}}\left(\frac{\lambda^3}{v}\right)^2 + \cdots$

(8.65) ort. işgal sayısı: $\langle n_p \rangle = \dfrac{z e^{-\beta\epsilon_p}}{1 + z e^{-\beta\epsilon_p}}$ $\ll 1$

Maxwell-Boltzmann formu $\longrightarrow$ $\simeq z e^{-\beta\epsilon_p} = \frac{\lambda^3}{v} e^{-\beta\epsilon_p}$

$$\frac{Pv}{kT} = \frac{f_{5/2}}{f_{3/2}} \qquad f_{5/2} = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{5/2}}$$

$$f_{3/2} = \sum_{l=1}^{\infty} \frac{(-1)^{l+1} z^l}{l^{3/2}}$$

$$= \frac{z - \frac{z^2}{2^{5/2}} + \cdots}{z - \frac{z^2}{2^{3/2}} + \cdots} = \left(1 - \frac{z}{2^{5/2}} + \cdots\right)\left(1 + \frac{z}{2^{3/2}} - \cdots\right)$$

$$= 1 + \frac{z}{2^{3/2}} - \frac{z}{2^{5/2}} + \cdots \qquad \frac{1}{\sqrt{2}}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4\sqrt{2}} = \frac{1}{2^{5/2}}$$

BS K $+\ \frac{z}{2^{5/2}} = 1 + \frac{1}{2^{5/2}}\left(\frac{\lambda^3}{v}\right) + \cdots$

Düşük T & Büyük n $\left(\lambda^3 / v \gg 1 \right)$

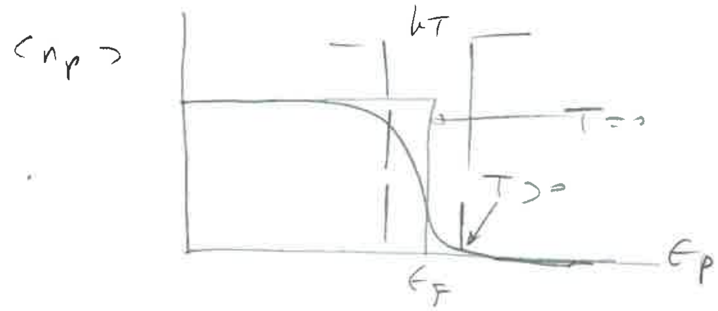
$$\frac{\lambda^3}{v} = \frac{4}{3\sqrt{\pi}} (\ln z)^{3/2} \qquad \lambda = \sqrt{\frac{2\pi\hbar^2}{mkT}} \text{ termal DB.}$$

$$\frac{1}{v} \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2} = \frac{4}{3\sqrt{\pi}} \left(\frac{\mu}{kT} \right)^{3/2}$$

$$\mu \to \epsilon_F \equiv \frac{\hbar^2}{2m} \left(\frac{6\pi^2}{v} \right)^{2/3}$$ FE mutlak sıfırda limit

$$\langle n_p \rangle = \frac{z e^{-\beta \epsilon_p}}{1 + z e^{-\beta \epsilon_p}} \qquad z = e^{\mu} \qquad \mu = \frac{\mu}{kT} = \frac{\epsilon_F}{kT}$$

$$= \frac{1}{z^{-1} e^{+\beta \epsilon_p} + 1} \cong \frac{1}{e^{+\beta(\epsilon_p - \epsilon_F)} + 1}$$



$$\langle n_p \rangle_{T=0} = \begin{cases} 1 & \epsilon_p < \epsilon_F \\ 0 & \epsilon_p > \epsilon_F \end{cases}$$

$$\frac{\lambda^3}{v} = f_{3/2}(z) = \frac{4}{3\sqrt{\pi}} (\underbrace{\ln z}_{\mu/kT})^{3/2} \left[1 + \frac{\pi^2}{8} (\ln z)^{-2} + \ldots \right]$$

$$\frac{1}{v} \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2} \frac{3\sqrt{\pi}}{4} = \mu^{3/2} \left[1 + \frac{\pi^2}{8} (\ln z)^{-2} + \ldots \right]$$

$$\left[\frac{1}{v} \frac{3\sqrt{\pi}}{4} \right]^{2/3} \frac{2\pi\hbar^2}{mkT} = \frac{\mu}{kT} \left[\qquad \right]^{2/3}$$

$$\Rightarrow \mu = \epsilon_F \left[1 + \frac{\pi^2}{8} (\ln z)^{-2} + \ldots \right]^{2/3}$$

BS K

$$\mu = \epsilon_F \left[1 - \frac{\pi^2}{8} \left(\frac{kT}{\epsilon_F} \right)^2 + \cdots \right]$$

$kT_F = \epsilon_F$ sıcaklığını tanımlayalım.

$T < T_F$ dejenere "bu durumda parçacıklar mümkün olan en düşük enerji düzeylerini işgal etmeye meyillidirler."

$T > T_F$ " olmayan

dejenerelik sıcaklığı

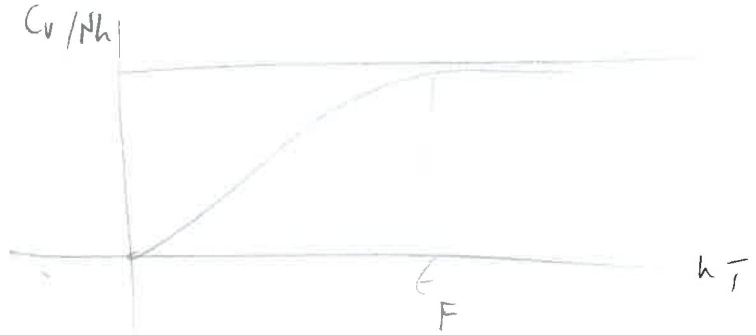
$$U = \sum_p \langle n_p \rangle \epsilon_p \to \int d^3p \, \langle n_p \rangle \epsilon_p = \frac{V}{h^3} \frac{4\pi}{2m} \int_0^\infty dp \, p^4 \langle n_p \rangle$$

$$= \frac{3}{2} NkT \frac{f_{5/2}(z)}{f_{3/2}(z)}$$

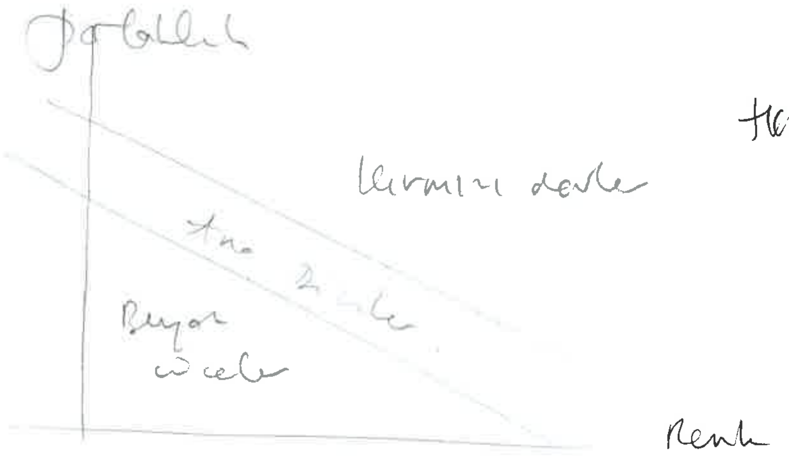
$$\left(\frac{\partial U}{\partial T} \right)_V = \frac{3}{5} N \epsilon_F \left[1 + \frac{5}{12} \pi^2 \left(\frac{kT}{\epsilon_F} \right)^2 + \cdots \right]$$

$$= \frac{V}{4\pi^2 m \hbar^3} \int_0^\infty dp \frac{p^5}{5} \left(- \frac{\partial}{\partial p} \langle n_p \rangle \right) = \frac{\beta V}{20\pi^2 m^2 \hbar^3} \int_0^\infty dp \frac{p^6 e^{\beta(\epsilon_p - \mu)}}{\left(e^{\beta(\epsilon_p - \mu)} + 1 \right)^2}$$

$$\frac{C_V}{Nk} \simeq \frac{\pi^2}{2} \frac{kT}{\epsilon_F} \qquad P = \frac{2}{3} \frac{U}{V}$$



11.2. Beyaz cücelerin teorisi :

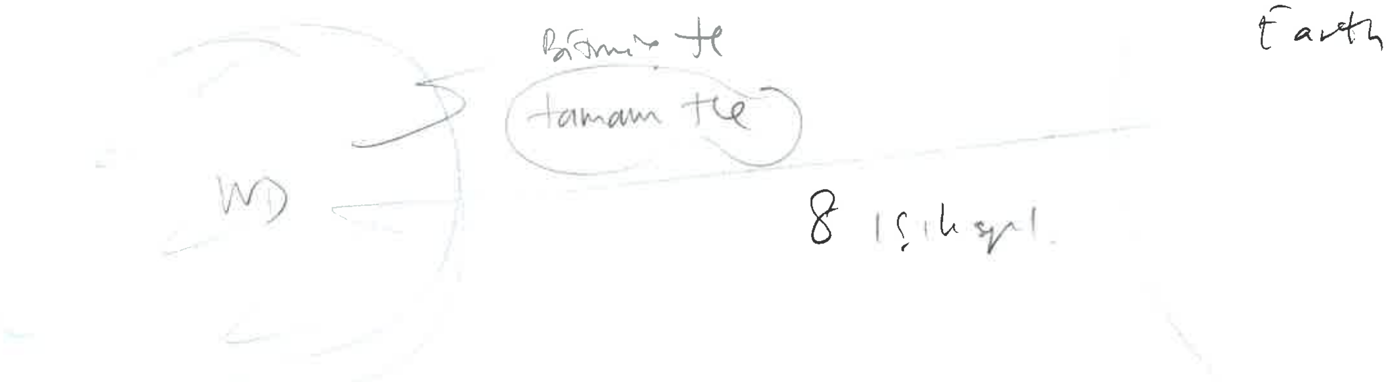


Hertzsprung-Russell diyagramı.

Merkez sıcaklık 10^7 °K

$\epsilon_F \sim 10^{11}$ °K çok

Düşük sıcaklık yüksek konsantrasyon.



Sirius

#1 $\epsilon_p = \sqrt{(pc)^2 + (mc^2)^2}$

$E_0 = 2 \sum_{|p|<p_F} \epsilon_p$ $\quad \langle n_p \rangle = 1$ sıfır sıcaklık

spin

$= 2 \frac{V}{h^3} \int^{p_F} dp \, 4\pi p^2 \sqrt{(pc)^2 + (mc^2)^2}$

BS K

$$\text{Yoğ.} \simeq 10^{7}\, g/cm^{3} \simeq 10^{7}\rho_{\odot}$$

$$\text{Kütle} \simeq 10^{33}\, g \simeq M_{\odot}$$ Güneş

$T_{\star}$ Merkezi sıcaklık $\simeq 10^{7} K \simeq T_{\odot}$

10^{3} eV ile atomlar tamamen iyonize

gaz $\equiv$ He çekirdeği + electronlar

= ideal Fermi gaz.

10^{30} elect / cm^{3}

$$\epsilon_F \simeq \frac{\hbar^2}{2m}\frac{1}{v^{2/3}} \simeq 20\, MeV \rightarrow T_F \simeq 10^{11} K \gg T_{\star}$$

$\Downarrow$

Dejenere Fermi Gazı

$\Downarrow$

Taban durumda.

#1 buraya! bay

$$\frac{V}{h^3}\left(\frac{4}{3}\pi p_F^3\right) = \frac{N}{2} \qquad \text{ya da} \qquad p_F = \hbar\left(\frac{3\pi^2}{v}\right)^{1/3}$$

$$x = p/m_e c \rightarrow \qquad E_0 = \frac{2V}{h^3} mc^2 \int dp\, 4\pi p^2 \sqrt{1+(\underbrace{p/mc}_{x})^2}$$

$$= \frac{2V}{h^3} m^4 c^5 4\pi \int_0^{x_F} dx\, x^2 \sqrt{1+x^2}$$

$$= \frac{V}{\hbar^3}\frac{m^4c^5}{\pi^2} \underbrace{\int_0^{x_F} dx\, x^2\sqrt{1+x^2}}_{f(x_F)}$$

BS K

$$f(x_F) = \begin{cases} \frac{1}{3} x_F^3 \left(1 + \frac{3}{10} x_F^2 + \cdots\right) & x_F \ll 1 \\ \frac{1}{4} x_F^4 \left(1 + \frac{1}{x_F^2} + \cdots\right) & x_F \gg 1 \end{cases}$$

$$M = N(m_e + 2m_p) \cong 2m_p N$$

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} \qquad \frac{V}{N} = v = \frac{8\pi}{3} m_p \frac{R^3}{M}$$

$$x_F = \frac{\hbar}{mc}\left(\frac{3\pi^2}{v}\right)^{1/3} \equiv \frac{\bar{M}^{1/3}}{\bar{R}} \qquad \bar{M} = \frac{9\pi}{8}\frac{M}{m_p}$$

$$\bar{R} = \frac{R}{\hbar/mc}$$

Compton

$$P_0 = -\frac{\partial E_0}{\partial V} = \frac{m^4c^5}{\hbar^3\pi^2}\left[-f(x_F) - v\frac{\partial f}{\partial x_F}\frac{\partial x_F}{\partial v}\right]$$

non-relat

$$P_0 = \left(\frac{m^4c^5}{15\hbar^3\pi^2}\right) x_F^5 = \frac{4}{5} K \frac{\bar{M}^{5/3}}{\bar{R}^5} \qquad x_F \ll 1 \text{ non rel}$$

$$P_0 = K\left(\frac{\bar{M}^{4/3}}{\bar{R}^4} - \frac{\bar{M}^{2/3}}{\bar{R}^2}\right) \qquad x_F \gg 1 \text{ extremely rel}$$

$$K = \frac{mc^2}{12\pi^2}\left(\frac{mc}{\hbar}\right)^3$$

$$-\int_0^R P_0 4\pi r^2 dr = +\alpha\gamma\frac{M^2}{R}$$

~ 1 G

$$P_0 = \frac{\alpha}{4\pi}\frac{\gamma M^2}{R^4} \sim \frac{\alpha}{4\pi}\gamma\left(\frac{8m_p}{9\pi}\right)^2\left(\frac{mc}{\hbar}\right)^4\frac{\bar{M}^2}{\bar{R}^4}$$

BS K

(a) $T \gg T_F$

$$P_0 = \frac{kT}{v} \quad , \quad v = \frac{8\pi}{3}\,\frac{m_p R^3}{M}$$

$$= \frac{3kT}{8\pi m_p}\,\frac{M}{R^3} = \frac{\alpha}{4\pi}\,\frac{\gamma M^2}{R^4}$$

$$R = \frac{2}{3}\alpha M\,\frac{m_p \gamma}{kT}$$

(b) $x_F \ll 1$ $\qquad \frac{4}{5} K \frac{\bar{M}^{5/3}}{\bar{R}^5} = K' \frac{\bar{M}^2}{\bar{R}^4}$

$$\bar{M}^{1/3}\,\bar{R} = \frac{4}{5}\,\frac{K}{K'}$$ yoq. dürül ile

(c) $x_F \gg 1$

$$P_0 = K\left(\frac{\bar{M}^{4/3}}{\bar{R}^4} - \frac{\bar{M}^{2/3}}{\bar{R}^2}\right) = K'\frac{\bar{M}^2}{\bar{R}^4}$$

$$\bar{R} = \bar{M}^{-1/3}\sqrt{1 - (\bar{M}/\bar{M}_0)^{2/3}} \qquad \bar{M}_0 \simeq M_\odot$$

$M_0 = 1.4\, M_\odot$ beyaz cüce şartı. Chandrasekhar 1937

BS K

Magnetik Özellikler

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 - \mu_B\, \vec{\sigma}\cdot\vec{H}$$

$$Q_N = \mathrm{Tr}\left(e^{-\beta\mathcal{H}}\right) \quad \text{ya da} \quad \mathcal{Z}(z,V,T) = \sum_{N=0}^{\infty} z^N Q_N$$

$$\frac{1}{V}\, kT \frac{\partial}{\partial H} \ln Q_N = \mathcal{M} \quad \text{ya da} \quad \mathcal{M} = kT \frac{\partial}{\partial H}\left(\frac{\ln \mathcal{Z}}{V}\right)$$

$$\chi \equiv \frac{\partial M}{\partial H} \begin{cases} < 0 & \text{diyamag.} \\ > 0 & \text{param.} \end{cases}$$

İlk önce ilk terim ile yetinip yörüngesel katkıyı bulacağız. bu Landau diyamagnetizması. Daha sonra 2. terimin katkısı ile Pauli paramagnetizmasını bulacağız.

Landau Diyamagnetizması

$$\mathcal{H} = \frac{1}{2m}\left(\bar{p} + \frac{e}{c}\bar{A}\right)^2 \qquad \bar{A}(\bar{r}) \to \bar{A}(\bar{r}) - \bar{\nabla}\omega(\bar{r}),$$

$$\psi(\bar{r}) \to \psi(\bar{r})\, e^{-ie\omega(\bar{r})/c}$$

$$A_x = -Hy \qquad A_y = A_z = 0$$

$\hat{z} \parallel \vec{H}$

$$\mathcal{H} = \frac{1}{2m}\left[\left(p_x - \frac{eH}{c} y\right)^2 + p_y^2 + p_z^2\right]$$

$$\mathcal{H}\psi = E\psi$$

$$\psi(r) = e^{ik_x x + ik_z z} f(y)$$

BS K

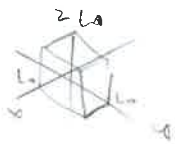
$$\left[\frac{p_y^2}{2m} + \frac{1}{2} m \omega_0^2 (y - y_0)^2 \right] f(y) = \varepsilon' f(y)$$

$$\omega_0 = \frac{eH}{mc} \qquad y_0 = \frac{\hbar c}{eH} k_x \qquad \epsilon' = \epsilon - \frac{\hbar^2 k_z^2}{2m}$$

k_x den bağımsız $\Rightarrow$ k_x in istediği değeri kadar dejenerelik var.

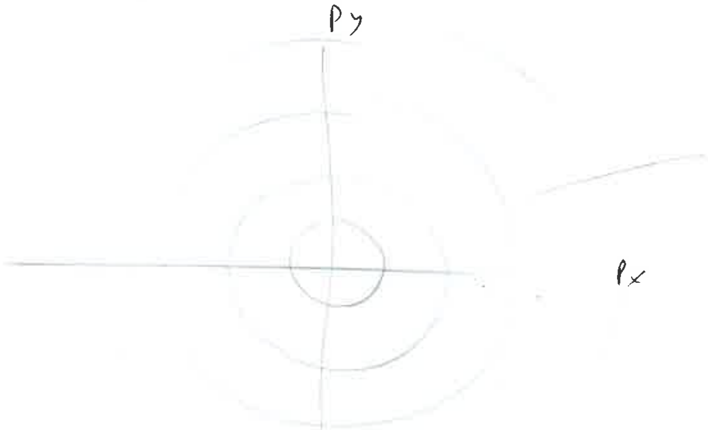
$$\epsilon = \frac{\hbar^2 k_z^2}{2m} + \hbar \omega_0 \left(j + \frac{1}{2} \right) \qquad 0, 1, 2, \ldots = j$$

$$\hbar k_z = p_z \qquad p_z = \frac{nh}{L} \qquad n = 0, 1, 2, \ldots$$



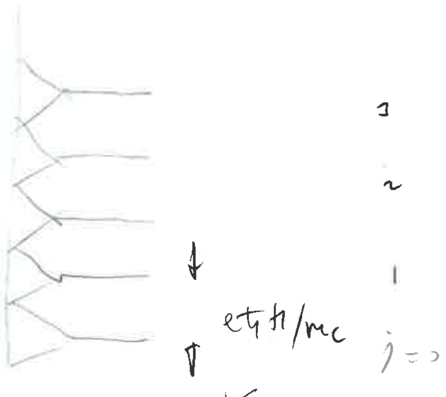
Sabit bir p_z için

$$\frac{p_x^2 + p_y^2}{2m} \longleftrightarrow \frac{e\hbar H}{m} \left(j + \frac{1}{2} \right)$$



$$\frac{p_\perp^2}{2m} = \frac{e\hbar H}{m}$$

$k_x L = 2\pi n_x$ desem $n_x = 0, \pm 1, \ldots$



$$\frac{eH}{hc} = \ell$$

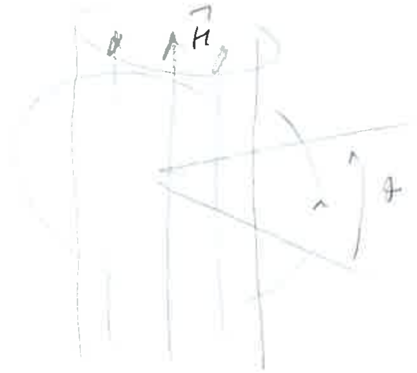
$$g = \frac{\ell}{L^2} \quad \text{dejenere}$$

$$g = \pi p_\perp^2 \left(\frac{L}{h} \right)^2 = \pi \frac{\cancel{2} e \hbar H}{c} \frac{L^2}{h^2}$$

$$= \frac{eH}{hc} L^2$$

BS K

Akı kuantumlanması :



e^-'nin olduğu yerde H yok. Ama, $A = \bar{\nabla}\omega$ var

$$\oint_C d\vec{\ell}\cdot\vec{A} = \int \bar{\nabla}\times\bar{A}\cdot d\vec{s}$$
$$= \int \bar{B}\cdot d\vec{s} = \Phi$$

$$\omega = \frac{\Phi\theta}{2\pi}$$

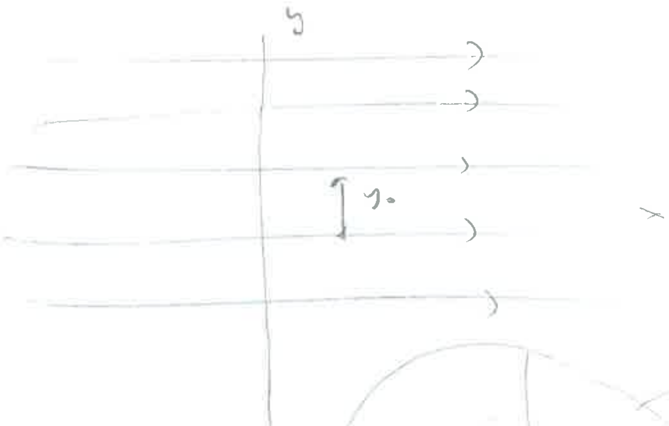
$$\bar{A} \to \bar{A} - \bar{\nabla}\omega$$

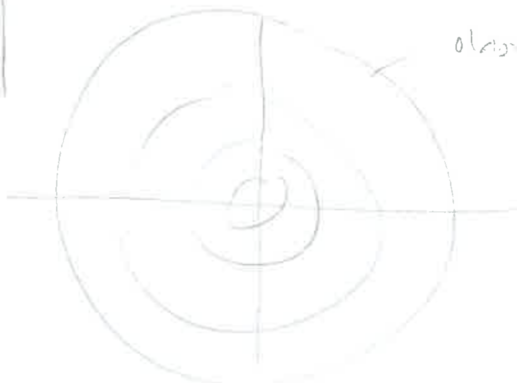
$$f = e^{-i\omega e/\hbar c} = e^{-i\theta\left(\frac{e\Phi}{\hbar c}\right)} \quad \text{tutarsızlık}$$

(a) e^- lokalize olsun deriz.

(b) e^- yörüngelerinin extended olduğunu söyleriz. Akı kuantumlanması

$$\Phi_0 = \frac{\hbar c}{e} \qquad \Phi = n\Phi_0 \qquad g = \frac{\Phi}{\Phi_0}$$





olasılığın max. olduğu çizgide bir Φ_0 muhafaza eder.

BS K

BS K

$$= \frac{2gL}{\lambda 2x} z \left(1 - x + \frac{x^2}{2} + \dots\right)\left(1 + x + \frac{2}{3}x^2 + \dots\right)$$

$$= \frac{2V}{\lambda^3}\left[1 - \frac{1}{24}\left(\frac{\hbar\omega_0}{kT}\right)^2\right]$$

$$M = \frac{\partial}{\partial H}\frac{\ln Z}{V} \Rightarrow \chi = \frac{\partial M}{\partial H}$$

$$\chi = -\frac{2}{3kT\lambda^3}\left(\frac{e\hbar}{2m}\right)^2$$

N hesaplanıp ve χ de konulur $\Rightarrow$ $\chi = -\frac{1}{3kTv}\left(\frac{e\hbar}{2mc}\right)^2$

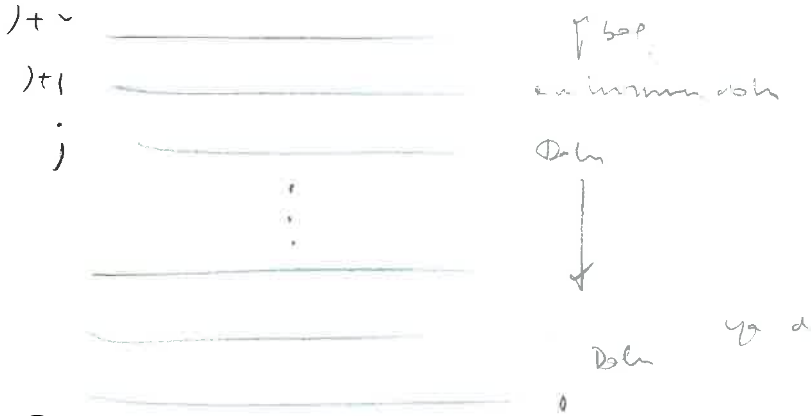
De Haas - Van Alphen Etkisi

H, $g \geq N$ olacak şekilde $\Rightarrow$ tüm parçacıklar en alt düzeyde bulunur $E_0 = N\frac{e\hbar H}{2mc}$

H, $g \leq N$ olacak şekilde $\Rightarrow$ bazı parçacıklar yüksek enerji düzeylerini doldurur.

$\epsilon = 2\mu_0 H (j + 1/2)$ $\mu_0 = e\hbar/2mc$ $g = NH/H_0$

$H_0 = n\hbar c/e$



H öyle olsun ki

$(j+1)g < N < (j+2)g$

ya da $j+1 < \frac{N}{g} < j+2$

$$\frac{1}{j+1} > \frac{H}{H_0} > \frac{1}{j+2} \qquad H_0 = \frac{nhc}{e} = \frac{hc}{e}\frac{N}{L^2}$$

sist. in energia $\quad E_0(H) = g \sum_{i=0}^{j} \epsilon_i + [N - (j+1)g]\, \epsilon_{j+1}$

$$= g\frac{e\hbar H}{mc}\sum_0^j \left(i+\frac{1}{2}\right) + \frac{e\hbar H}{mc}\left\{N - (j+1)g\left(j+\frac{3}{2}\right)\right\}$$

$$= \frac{e\hbar H}{mc}\left[\frac{g}{2}(j+1)^2 + \{N - (j+1)g\}\left(j+\frac{3}{2}\right)\right]$$

$$= \frac{e\hbar H}{mc}\left\{N\left(j+\frac{3}{2}\right) + g\left[\frac{1}{2}(j+1)^2 - (j+1)\left(j+\frac{3}{2}\right)\right]\right\}$$

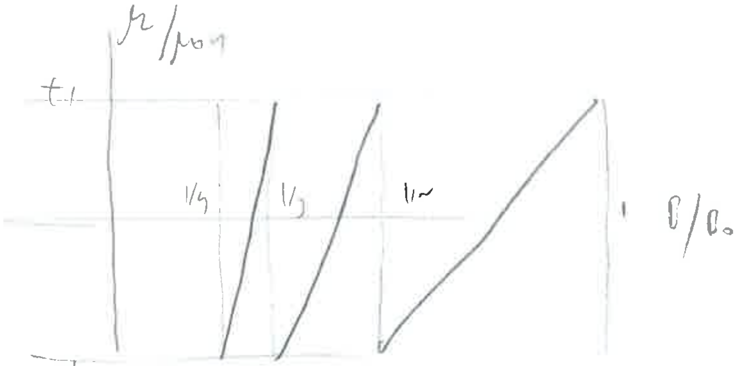
$$-\frac{j+2}{2}(j+1)$$

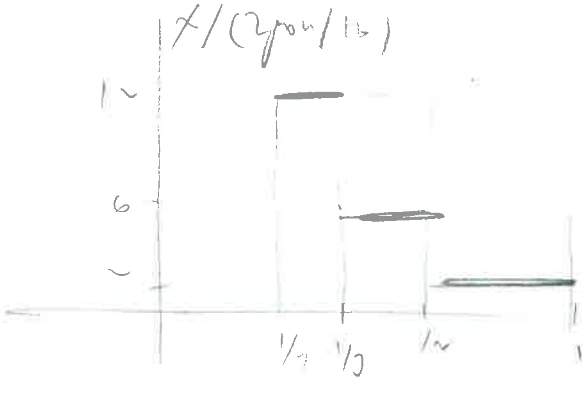
$$= \frac{e\hbar H}{mc}\left\{N\left(j+\frac{3}{2}\right) - \frac{\overset{NH/H_0}{g}}{2}(j+1)(j+2)\right.$$

$$\frac{E_0}{N} = \begin{cases} \frac{e\hbar H}{mc}\left\{\left(j+\frac{3}{2}\right) - \frac{1}{2}(j+1)(j+2)\frac{H}{H_0}\right. & \frac{1}{j+1} > x > \frac{1}{j+2} \\ \frac{e\hbar H_0 x}{2mc} & x = \frac{H}{H_0} \end{cases}$$

$$Q_N = \sum e^{-\beta E(j)} \qquad kT\,\partial_H \ln Q_N$$

$$\mu = \begin{cases} -\mu_0 n & x > 1 \\ \mu_0 n\left[2(j+1)(j+2)x - (2j+3)\right] & \frac{1}{j+2} < x < \frac{1}{j+1} \end{cases}$$





BS K

II.b. Pauli Paramagnetizması:

diamagnetic

paramag.

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 - \mu_0 \vec{\sigma}\cdot\vec{H}$$

$\vec{\sigma}\cdot\vec{H} = sH = \pm H$

$$\mathcal{H} = \frac{\vec{p}^2}{2m} - \mu\,\vec{\sigma}\,\vec{H} \qquad \text{olsun.}$$

$$\epsilon_{\vec{p},s} = \frac{p^2}{2m} - s\mu_0 H$$

N-parcacıklı dist. in enerji özdeğeri $n_{\vec{p},s}$ ile etiketlenir

$$E_n = \sum_{\vec{p}}\sum_{s} \epsilon_{\vec{p},s}\, n_{\vec{p},s} = \sum_{\vec{p}}\left[\left(\frac{\vec{p}^2}{2m} - \mu_0 H\right) n_{\vec{p},+1} + \left(\frac{\vec{p}^2}{2m} + \mu_0 H\right) n_{\vec{p},-1}\right]$$

$$n_{\vec{p},s} = 0,1 \qquad \sum_{\vec{p}s} n_{\vec{p},s} = N$$

$$n_{\vec{p}+} = n_p^+ \qquad \sum n_p^+ = N^+$$

$$n_{p-} = n_p^- \qquad \sum n_p^- = N^-$$

$$E_n = \sum_p \left(n_p^+ + n_p^-\right)\frac{p^2}{2m} - \mu_0 H\left(N_+ - N_-\right)$$

$$Q_N = \sum_{\{n_p^+\}\{n_p^-\}} \exp\left\{-\beta\sum_p \left(n_p^+ + n_p^-\right)\frac{p^2}{2m}\right\}\exp\left\{-\beta\mu_0 H\left(N_+ - N_-\right)\right\}$$

$$= \sum_{N^+=0}^{N} e^{\beta\mu_0 H(2N_+ - N)} \sum_{n_p^+}{}'' e^{-\beta\sum_p \frac{p^2}{2m} n_p^+} \sum_{n_p^-}{}''' e^{-\beta\sum_p \frac{p^2}{2m} n_p^-}$$

$$\sum_{\vec{p}}{}'' n_p^+ = N_+ \qquad \sum_{\vec{p}}{}''' n_p^- = N_- = N - N_+$$

BS K

$$Q_N^{(0)} \equiv \sum_{\sum n_p = N} e^{-\beta \sum \frac{p^2}{2m} n_p} \equiv e^{-\beta A(N)}$$

$$Q_N = e^{-\beta \mu_0 H N} \sum_{N_+=0}^{N} e^{2\beta\mu_0 H N_+} Q_{N_+}^{(0)} Q_{N-N_+}^{(0)}$$

$$\frac{1}{N} \log Q_N = -\beta\mu_0 H + \frac{1}{N} \ln \sum e^{2\beta\mu_0 H N_+} e^{-\beta A(N_+)} e^{-\beta A(N-N_+)}$$

$$\frac{1}{N} \log Q_N = \beta f(\bar{N}_+)$$

$$f(N_+) = \mu_0 H \left(\frac{2N_+}{N} - 1\right) - \frac{1}{N}\left[A(N_+) + A(N-N_+)\right]$$

extremum for $\bar{N}_+$ 'ys bud.

$$\mathcal{M} = \frac{\mu_0 (2\bar{N}_+ - \bar{N}_-)}{V}$$

$$\left.\frac{\partial f(N_+)}{\partial N_+}\right|_{N_+=\bar{N}_+} = 0 \qquad 2\mu_0 H - \left[\frac{\partial A(N_+)}{\partial N_+} + \frac{\partial A(N-N_+)}{\partial N_+}\right]_{\bar{N}_+} = 0$$

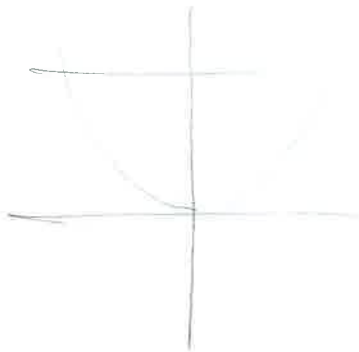
$$kT\left[\nu(\bar{N}_+) - \nu(N - \bar{N}_+)\right] = 2\mu_0 H$$

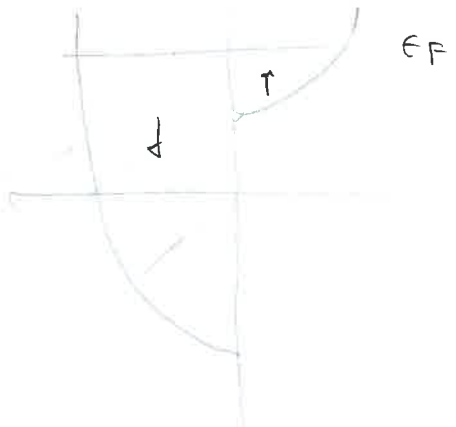
2 word!

$$\epsilon_F(N) \equiv \left(\frac{3\pi^2 N}{V}\right)^{2/3} \frac{\hbar^2}{2m}$$

$$kT\nu(N) = \epsilon_F(2N)\left\{1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F(2N)}\right)^2 + \dots\right\}$$

BS K





$$\epsilon_F(2\bar N_+) - \epsilon_F(2N - 2\bar N_+) - \frac{\pi^2}{12}(kT)^2\left\{\frac{1}{\epsilon_F(2\bar N_+)} - \frac{1}{\epsilon_F(2N-2\bar N_+)}\right\} + \dots$$

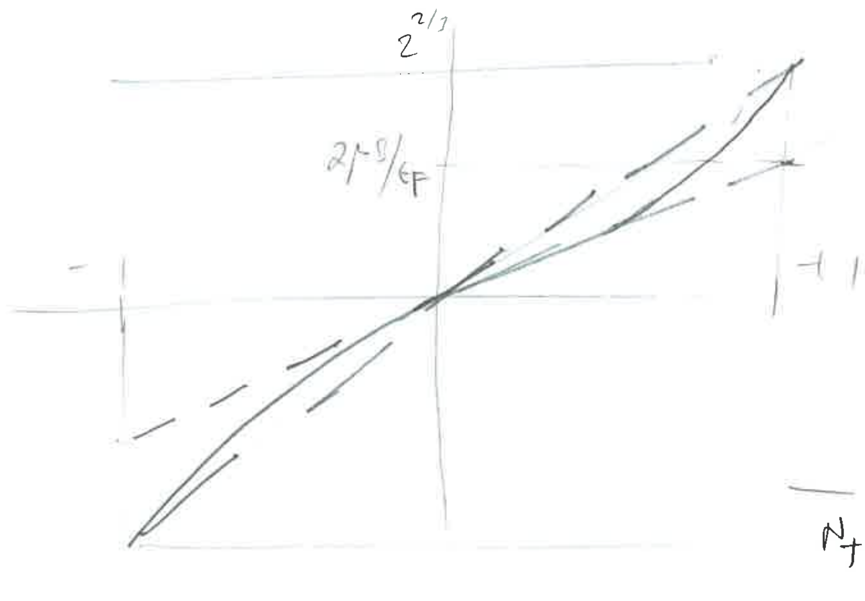
$$= 2\mu_0 H$$

$$r \equiv \frac{2\bar N_+}{N} - 1 \qquad -1 \le r \le +1$$

$$(1+r)^{2/3} - (1-r)^{2/3} - \frac{\pi^2}{12}(kT)^2\frac{1}{\epsilon_F^2}\left[(1+r)^{-2/3} - (1-r)^{-2/3}\right] = 2\mu_0 H/\epsilon_F$$

✓ $T \to 0$

$$(1+r)^{2/3} - (1-r)^{2/3} = \frac{2\mu_0 H}{\epsilon_F}$$ kan grafisch slecht ontwikkelen [uncertain reading]



$$H \ll \frac{\epsilon_F}{2\mu}$$

$$\left(1+\frac{2r}{3}\right) - \left(1-\frac{2r}{3}\right) = \frac{2\mu_0 H}{\epsilon_F}$$

$$r = \frac{3\mu_0 H}{2\epsilon_F}$$

$$\bar N_+ = \frac{N}{2}\left(1 + \frac{3\mu_0 H}{2\epsilon_F}\right)$$

$$\mathcal{M} = \frac{\mu_0 r}{v} = \frac{3\mu_0^2 H}{2\epsilon_F v} \qquad \chi = \frac{3\mu_0^2}{2\epsilon_F v}$$

$0 < kT \ll \epsilon_F \qquad \mu_0 H \ll \epsilon_F$

$$r = \frac{3\mu_0 H}{2\epsilon_F}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\right]$$

$$\chi \doteq \frac{3\mu_0^2}{2\epsilon_F v}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\right]$$

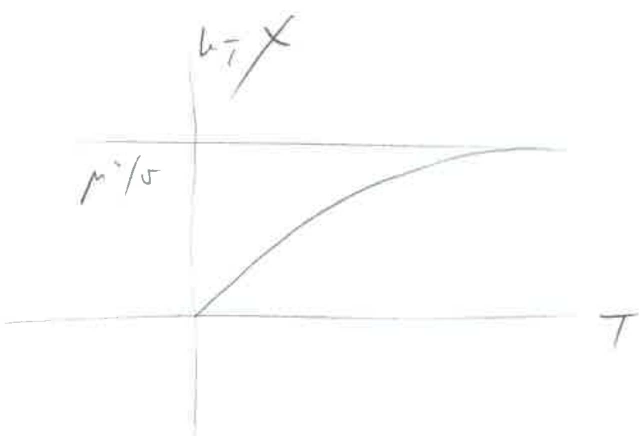
$kT \gg \epsilon_F \qquad z = \frac{\lambda^3}{v} + \frac{1}{2^{3/2}}\left(\frac{\lambda^3}{v}\right)^2 + \cdots \qquad \ln z = \frac{\mu}{kT}$

$$\mu(N) = \ln\left(\frac{N\lambda^3}{v}\right)$$

$$\ln\left(\frac{\lambda^3(1+r)}{v}\right) - \ln\left(\frac{\lambda^3(1-r)}{v}\right) = \frac{2\mu_0 H}{kT}$$

$$\frac{1+r}{1-r} = \frac{v}{\lambda^3}\, e^{2\mu_0 H/kT}$$ [illegible]

$$\chi \simeq \frac{\mu_0^2}{kTv}$$



11.7 Mükemmel olmayan bir gazın magnetik özellikleri

Parçacıklar arası etkileşme bir elektron gazının magnetik özelliklerini nasıl değiştirir?

İtici bir etkileşme → paramagnetizmayı arttırır, çünkü, iki elektron itici enerjiyi minimize edebilmek için uzaysal antisim. dalga fonk.'unu tercih edecektir. Ama antisim. uzaysal dalga fonk.u simetrik spin dalga fonk.'u gerektirir (triplet durum) böylelikle itme elektronların spinlerini hizalar.

1/2 spinli Fermi gazı alalım. Bunlar a saçılma uzunluğu ile belirlenmiş olsun:

e-e etk.

$$E_N = \sum_p (n_p^+ + n_p^-)\frac{p^2}{2m} + \frac{4\pi a\hbar^2}{mV} N_+ N_- - (N_+ - N_-)\mu_0 H$$

bunun geçerlilik şartı $k_F |a| \ll 1$ $\quad k_F^2 = (3\pi^2 n)^{2/3}$ $\quad na^3 \ll 1$

$$Q_N = \sum_{\{n_p^+\}\{n_p^-\}} \exp\left\{ \sum_p (n_p^+ + n_p^-)\frac{p^2}{2m} - \mu_0 H (N_+ - N_-) + \frac{4\pi a\hbar^2}{mV} \right\}$$

$$\frac{1}{N}\ln Q_N = \beta\, g(\bar{N}_+) \qquad g(\bar{N}_+) : \text{maks}\,(g(N_+))$$

f için $\longrightarrow$ $f(\bar{N}_+) : \text{min}\,(f(N_+))$ dir.

$$g(N_+) = \mu_0 H \left(\frac{2N_+}{N} - 1 \right) - \frac{4\pi a \hbar^2}{mV} N_+ (N - N_+)$$

$$- \frac{1}{N} \left[A(N_+) + A(N - N_+) \right]$$

$$\left. \frac{\partial g(N_+)}{\partial N_+} \right|_{N_+ = \bar{N}_+} = 0 \qquad \left. \frac{\partial^2 g(N_+)}{\partial N_+^2} \right| < 0$$

denk. mi kökleri $\bar{N}_+$

$$\exists \begin{cases} kT\left[\nu(\bar{N}_+) - \nu(N - \bar{N}_+) \right] = 2\mu_0 H + \frac{4\pi a \hbar^2}{mV} (2\bar{N}_+ - N) \\ kT\left[\nu'(\bar{N}_+) + \nu'(N - \bar{N}_+) \right] - \frac{8\pi a \hbar^2}{mV} > 0 \end{cases}$$

$$\nu'(N) \equiv \partial \nu(N) / \partial N$$

$$r \equiv \frac{2\bar{N}_+}{N} - 1 \quad \text{olsun} \quad -1 \leq r \leq +1 \qquad \lambda = \sqrt{2\pi \hbar^2 / mkT}$$

o zaman

$$\begin{cases} kT \left\{ \nu\left[\frac{N}{2}(1+r) \right] - \nu\left[\frac{N}{2}(1-r) \right] \right\} = 2\mu_0 H + \frac{a\lambda^2}{v} 2kTr \\ \frac{\partial}{\partial r} \left\{ \nu\left(\frac{N}{2}(1+r) \right) - \nu\left(\frac{N}{2}(1-r) \right) \right\} - \frac{2a\lambda^2}{v} > 0 \end{cases}$$

$$\begin{cases} {}^{*1}\; kT\nu\left(\frac{N}{2}x \right) \simeq x^{2/3} \epsilon_F \left[1 - \frac{\pi^2}{12} \left(\frac{kT}{\epsilon_F} \right)^2 \frac{1}{x^{4/3}} \right] & \frac{kT}{\epsilon_F} \ll 1 \\ {}^{*2}\; kT\nu\left(\frac{N}{2}x \right) \simeq \ln \frac{\lambda^2}{2v} & kT/\epsilon_F \gg 1 \end{cases}$$

*2 (High T & low n $\lambda^3/v \ll 1$) $z = \frac{\lambda^3}{v} = \frac{1}{2^{3/2}} \left(\frac{\lambda^3}{v} \right)^2 + \ldots$

*1 (low T & high n $kT\nu = kT \ln z = \epsilon_F \left(1 - \frac{\pi^2}{12} \left(\frac{kT}{\epsilon_F} \right)^2 + \ldots \right)$

BS K

kendiliğinden mıknatıslanma:

$H=0$ durumunu alalım. $T=0$ dahi durumu bakalım.

$$\nu\left(\frac{N}{2}(1+r)\right)-\nu\left(\frac{N}{2}(1-r)\right)=\frac{2a\lambda^2}{V}r$$

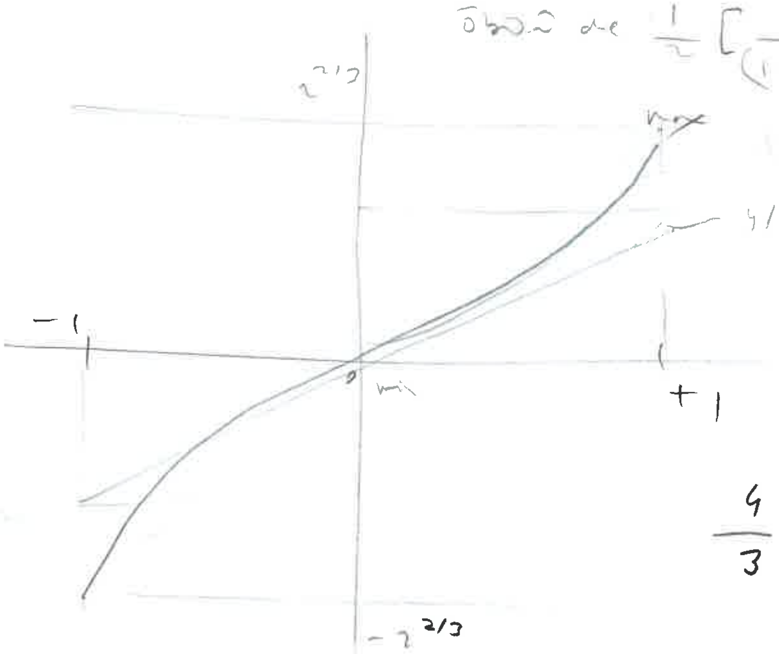
$$\frac{\epsilon_F}{\cancel{kT}}\left[(1+r)^{2/3}-(1-r)^{2/3}\right]=\frac{2a}{V}\frac{2\pi\hbar^2}{\cancel{mkT}}r$$

$$(1+r)^{2/3}-(1-r)^{2/3}=\frac{4\pi a\hbar^2}{Vm\epsilon_F}r \qquad \epsilon_F=\frac{\hbar^2k^2}{2m}$$

$$=\frac{8\pi a}{Vk_F^2}r=\frac{8\pi a k_F}{\cancel{k}\,3\pi^2/\cancel{k}}=\frac{8}{3\pi}ak_F r=\xi r$$

ξ

özüm de $\frac{1}{2}\left[\frac{1}{(1+r)^{1/3}}+\frac{1}{(1-r)^{1/3}}\right]>\frac{3}{4}\xi$



$$r=\frac{2N_+}{N}-1$$

$$\frac{4}{3}<\xi<2^{2/3}$$

$r=0$ fekle karın. $\Rightarrow$ $\xi<4/3$ Kmğ. yok.

$0<r<1$ $2^{2/3}>\xi>\frac{4}{3}$ kısmen var

$r=1$ $\Rightarrow$ $\xi>2^{2/3}$ tam mg. var.

itme yeterince güçlü ise $\rightarrow$ ferro

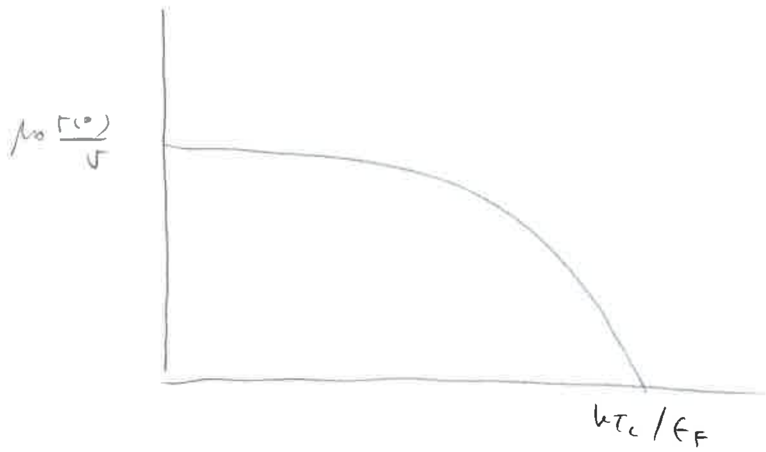
$$\zeta = \frac{\cancel{8} k_{F1}}{\cancel{3\pi}} = \frac{4}{3} \Rightarrow k_F = \frac{\pi}{2}$$ feromag. nın başladığı limit.

Sonlu sıcaklıklarda $T \neq 0$

$$(1+r)^{2/3} - (1-r)^{2/3} - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\left[\frac{1}{(1+r)^{2/3}} - \frac{1}{(1-r)^{2/3}}\right] = \zeta r$$

Eğer $r(0) = 0 \Rightarrow r(T) = 0$ olur.

" $r(0) \neq 0 \Rightarrow r(T) < r(0)$ sıcaklık arttıkça ferromagnetik-lik yok olur.



$$\frac{kT_c}{\epsilon_F} = \frac{3}{\pi}\sqrt{\zeta - \frac{4}{3}}$$

alfa << 1 olduğunda geçerli olan model genel bir model değil.

$\mathcal{H} \neq 0$ olduğu durum (paramagnetik alınganlık)

$$r = r_0(T) + \frac{\chi V}{\mu_0}\mathcal{H} - \text{küçük}$$

bunu 11.134 de koy!

$$\nu\left[\frac{N}{2}\left(1 + r_0 + \frac{\chi V}{\mu_0}\mathcal{H}\right)\right] - \nu\left[\frac{N}{2}\left(1 - r_0 - \frac{\chi V}{\mu_0}\mathcal{H}\right)\right] = \frac{2ad}{V}\left(r_0 + \frac{\chi V \mathcal{H}}{\mu_0}\right) + 2\mu_0 \mathcal{H}$$

$$\chi = (2\mu_0^2/v\epsilon_F) \Big/ \left\{ \frac{NkT}{2\epsilon_F} \left[\left[v'\left[\frac{N}{2}(1+r_0)\right] + v'\left[\frac{N}{2}(1-r_0)\right] \right]\right] - \frac{8k_Fa}{3\pi} \right\}$$

$$\chi \xrightarrow[T\to 0]{} \frac{3\mu_0^2/\epsilon_F v}{(1+r_0)^{-1/3} + (1-r_0)^{-1/3} - \frac{4}{\pi}k_Fa} > 0$$

$$\xrightarrow[T\to\infty]{} \frac{\mu_0^2}{kTv}$$

paramag. veya ferro mag. verir.

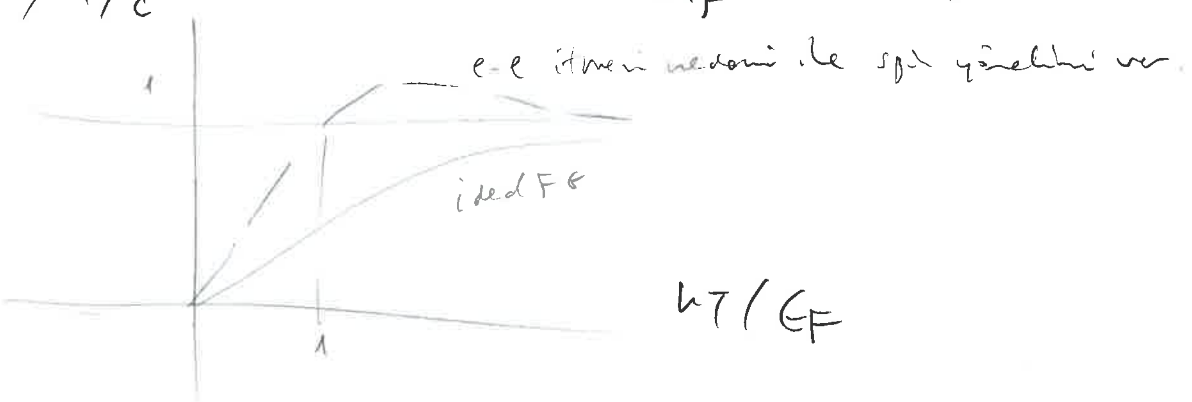
$r_0 = 0$ alıp paramanyetik durumu bakalım.

$$\chi = \frac{2\mu_0^2/\epsilon_F v}{\frac{NkT}{2\epsilon_F} v' - \frac{8k_Fa}{3\pi}}$$

$$\frac{T\chi}{C} = \frac{3k}{\epsilon_F} \frac{1}{f - \frac{2k_Fa}{\pi}}$$

$$f = \frac{3kT}{2\epsilon_F} \frac{N}{2} v'\left(\frac{N}{2}\right)$$

$\chi T/c$



$$\frac{\partial}{\partial T}\left(\frac{\chi T}{c}\right)\Big|_{T=0} = \frac{3}{2} \quad \text{ideal FG.}$$

BS K

11.3. $\epsilon_p = \frac{p^2}{2m} + mc^2$ $\quad mc^2 = 10^3$ MeV

$$\mathcal{Z} = \sum z^N Q_N \qquad \sum n_\lambda = N$$ bez warunku na
ilość cząstek.

0 gaza

$$Q_N = \sum_{\{n_p\}} e^{-\beta \sum \epsilon_p n_p}$$

$$= \sum_{n_0} e^{-\beta \epsilon_0 n_0} \sum_{n_1} e^{-\beta \epsilon_1 n_1} = \prod_p \left(1 + e^{-\beta \epsilon_p}\right)$$

$$\ln Q_N = \sum_p \left(1 + e^{-\beta \epsilon_p}\right)$$

$$= \frac{V}{h^3} 4\pi \int p^2 dp \left(1 + e^{-\beta \epsilon_p}\right)$$

$$U = -\frac{\partial \ln Q_N}{\partial \beta} = \frac{V 4\pi}{h^3} \int p^2 dp \frac{\epsilon_p e^{-\beta \epsilon_p}}{1 + e^{-\beta \epsilon_p}}$$

$$= \frac{V 4\pi}{h^3} \int p^2 dp \frac{\epsilon_p}{1 + e^{\beta \epsilon_p}}$$

$$p = \sqrt{2m(\epsilon_p - mc^2)} \qquad \frac{p}{m} dp = d\epsilon_p$$

$$\frac{U}{V} = \int u(\epsilon_p) d\epsilon_p \Leftarrow U = \frac{V 4\pi}{h^3} \int \frac{m\left[2m(\epsilon_p - mc^2)\right]^{1/2}}{\underbrace{1 + e^{\beta \epsilon_p}}} \epsilon_p d\epsilon_p$$

minimum kinetycznej
energii cząstki

$$\bar{N} = -\frac{1}{\beta} \frac{\partial \ln Q_N}{\partial \epsilon_p}$$

BS K

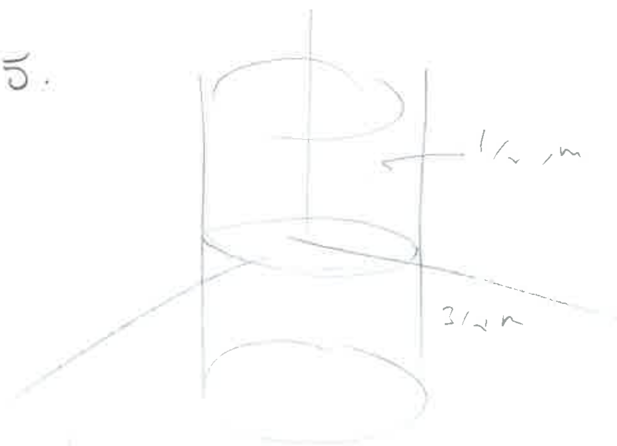
$$= \frac{V 4\pi}{h^3} \int p^2 dp \frac{1}{1+e^{-\beta \epsilon_p}}$$

$$= \frac{4\pi V}{h^3} \int [2m(\epsilon_p - mc^2)]^{3/2} \frac{1}{1+e^{\beta \epsilon_p}} d\epsilon_p$$

$$\frac{\bar{N}}{V} = \int \underbrace{(\quad)}_{\text{parçacık yoğ.}} d_p \qquad n(\epsilon_p, T) = \frac{4\pi}{h^3} \frac{(2m(\epsilon_p - mc^2))^{3/2}}{1+e^{\beta \epsilon_p}}$$

$\epsilon \to \infty$ için $n \to 0$ çıkar onun için parçacık sayısı korunmalı ve z eklenmelidir.

11. 5.



$T = 0$ ve ∞ da iki gazın bağıl yoğunlukları ?

Balonçlar ayrı.

$N = N_{1/2} + N_{3/2}$

$$\frac{PV}{kT} = \ln \mathcal{Z}(z, V, T) = \frac{LV}{(2\pi\hbar\lambda)^2} \sum (-1)^{\ell+1} \frac{z^\ell}{\ell^{3/2}} \qquad z \ll 1 \text{ için}$$

$$\frac{1}{v} = \frac{L}{2\pi\hbar\lambda^2} \sum_{\ell=1}^{\infty} (-1)^{\ell+1} \frac{z^\ell}{\ell^{3/2}} = \frac{L}{2\pi\hbar\lambda^2} \ln(1+z)$$

$$N = \sum_p \frac{1}{z^{-1} e^{\beta \epsilon_p} + 1} = \frac{V}{(2\pi\hbar)^3} \int d^3p \frac{1}{z^{-1} e^{\beta \epsilon_p} + 1} \qquad z^{-1} = e^{-\nu}$$

$$\frac{1}{v} = \frac{L}{2\pi\hbar\lambda^2} \ln z \quad \text{büyük } z\text{'ler için}$$

$$\langle n_p \rangle = \frac{1}{e^{\beta(\mu - \epsilon_p)}} \Bigg|_{T=0} = \begin{cases} 1 & 0 < \epsilon_F - \epsilon_p \\ 0 & 0 > \epsilon_F - \epsilon_p \end{cases}$$

BS K

$$N = g \sum_p \langle n_p \rangle = g \frac{V}{h^3} 2\pi L \int p\,dp \langle n_p \rangle$$

fil. hoord.

$$N = \frac{gV2\pi L}{h^3} \frac{m p_F^2}{2m} \epsilon_F$$

$$\epsilon_F = \frac{n \hbar^3}{2\pi L m g}$$

$$U = g \sum_p \epsilon_p \langle n_p \rangle = \frac{1}{2} N \epsilon_F$$

$$P = \frac{2}{3} \frac{U}{V} = \frac{1}{3} n \epsilon_F \qquad S = 1/2 \Rightarrow \quad g = 2S+1 = 2$$

$$P_{1/2} = \frac{(2\pi\hbar)^3}{12\pi m L_1} n_{1/2}^2$$

$$P_{3/2} = \frac{(2\pi\hbar)^3}{24\pi m (L - L_1)} n_{3/2}^2$$

aengaae $n_{1/2} = \sqrt{2\left(\frac{L}{L_1} - 1\right)}\; n_{3/2} \qquad T \to 0$

$T \to \infty \qquad \lambda^2 \to 0$

$$\frac{1}{v} = \frac{L}{2\pi\hbar \lambda^2}\left(z - \frac{z^2}{2} + \dots\right)$$

$\lambda^2 \to 0 \Rightarrow z$ h.ũil $\qquad z = \dfrac{2\pi\hbar\, \lambda^2/L}{v}$

$$\langle n_p \rangle = \frac{z e^{-\beta \epsilon_F}}{z e^{-\beta \epsilon_p} + 1} \simeq z e^{-\beta \epsilon_p}$$

$U = \frac{1}{2} N \epsilon_F \Rightarrow P = \frac{2U}{3V} = \frac{1}{3} n \epsilon_F \Big\} \qquad P_{1/2} = \frac{1}{3} n_{1/2}\left(\frac{kT}{2}\right) \qquad P_{3/2} = \frac{1}{3} n_{3/2}\left(\frac{kT}{4}\right)$

BS K

11.7.

$$E_n = \sum_p (n_p^+ + n_p^-)\frac{p^2}{2m} + \frac{4a\hbar^2}{mV} N_+(N-N_+) - (N_+ - N_-)\mu_0 H$$

$$\frac{1}{N}\ln Q_N = \beta g(\bar{N}_+)$$

$$g(\bar{N}_+) = \max g(N_+)$$

$$C_V = \left(\frac{\partial u}{\partial T}\right)_V \qquad U = -\frac{1}{\beta}\ln Q_N$$

$$\frac{U}{N} = -g(\bar{N}_+) - \beta\frac{\partial g(\bar{N}_+)}{\partial \beta}$$

$$\frac{2N_+}{N} - 1 = r$$

$$\frac{U}{N} = -g(\bar{N}_+) - \beta\frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}\frac{\partial \bar{N}_+}{\partial \beta}$$

$$= \frac{\partial \bar{N}_+}{\partial r}\frac{\partial r}{\partial T}\frac{\partial T}{\partial \beta} = \frac{N}{2}\frac{\partial r}{\partial T}kT^2$$

$$\frac{U}{N} = -g(\bar{N}_+) + T\frac{N}{2}\frac{\partial r}{\partial T}\frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}$$

$$\frac{C_V}{N} = -\frac{\partial g(\bar{N}_+)}{\partial T} + \frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}\frac{N}{2}\frac{\partial r}{\partial T} + T\frac{N}{2}\frac{\partial^2 r}{\partial T^2}\frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}$$

$$+ \frac{N}{2}\frac{\partial r}{\partial T}\frac{\partial}{\partial T}\left(\frac{\partial g(N_+)}{\partial \bar{N}_+}\right) = T\frac{\partial^2 g(\bar{N}_+)}{\partial T^2}$$

$$= \frac{\partial}{\partial T}\left[\frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}\frac{\partial \bar{N}_+}{\partial r}\frac{\partial r}{\partial T}\right]$$

BS K

$$= \frac{\partial^2 \bar{N}_+}{\partial T^2} \frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+} + \frac{\partial \bar{N}_+}{\partial T} \frac{\partial}{\partial T} \frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+}$$

$$\frac{\partial^2 \bar{N}_+}{\partial T^2} = \frac{\partial \bar{N}_+}{\partial r} \frac{\partial^2 r}{\partial T^2}$$

$$\frac{C_V}{N} = \frac{N}{2} T \frac{\partial}{\partial T}\left[\frac{\partial r}{\partial T} \frac{\partial}{\partial \bar{N}_+} g(\bar{N}_+) \right]$$

$$g(\bar{N}_+) = \mu_0 H \left(\frac{2\bar{N}_+}{N} - 1 \right) - \frac{4\pi a \hbar^2}{mV} \bar{N}_+ (N - \bar{N}_+) - \frac{1}{N}\left[A(\bar{N}_+) + A(N - \bar{N}_+) \right]$$

$$\frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+} = \mu_0 H \frac{2}{N} - \frac{4\pi a \hbar^2}{mV} \underbrace{(-2\bar{N}_+ + N)}_{I(r)} - \frac{1}{N} kT\, I(r)$$

$$\frac{C_V}{N} = \frac{N}{2} T \frac{\partial}{\partial T} \left(\frac{\partial r}{\partial T} \frac{\partial g(\bar{N}_+)}{\partial \bar{N}_+} \right)$$

$$= \frac{N}{2} T \left\{ \left[\frac{\partial^2 r}{\partial T^2} \left(\mu_0 H \frac{2}{N} - \frac{4\pi a \hbar^2}{mV} N r - \frac{1}{N} kT\, I(r) \right) \right] \right.$$

$$\left. + \frac{\partial r}{\partial T} \left(\frac{4\pi a \hbar^2}{mV} N \frac{\partial r}{\partial T} - \frac{1}{N} kT \frac{\partial I}{\partial T} + I(r) \right) \right\}$$

BS K