(EK)

KESİRLİ İSTATİSTİK

Wilczek Modeli

Fizikte özdeş parçacıklı çok cisimli sistemler ile uğraşan iki istatistik vardır.

$$\psi_S(\vec{r}_1,\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots,\vec{r}_N)$$
$$= \psi_S(\vec{r}_1,\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots,\vec{r}_N) \quad \text{BE}$$

$D=3$

$$\psi_A(\vec{r}_1,\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots,\vec{r}_N)$$
$$= -\psi_A(\vec{r}_1,\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots,\vec{r}_N) \quad \text{FD}$$

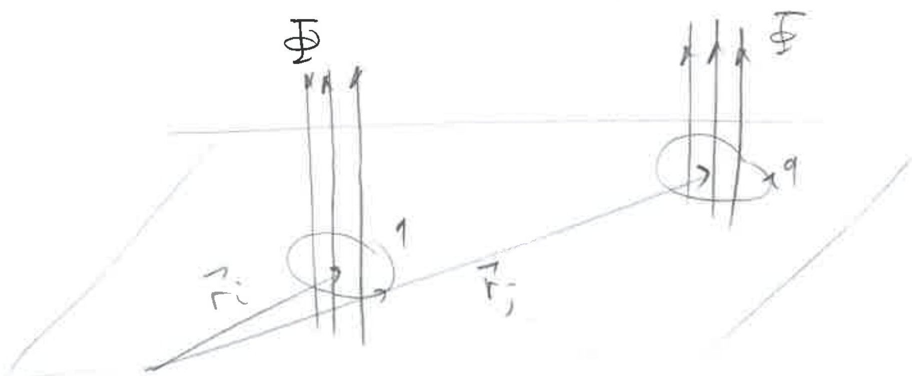
$$\psi_F(\vec{r}_1,\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots,\vec{r}_N) = e^{i\theta}\,\psi_F(\vec{r}_1,\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots,\vec{r}_N)$$

$$\theta = \begin{cases} 0 \quad (2\pi) & \text{BE} \\ \pi & \text{FD} \end{cases}$$

arasında kesirli istatistik ya da θ istatistiği diyoruz.

WM

parçacık = q + buna iliştirilmiş mag. akı Φ



BS K

konumları değiş tokuş edildiğinde, 180° dönmeye karşılık gelir, (saat yönü ya da tersi yönde) Her biri yolun zıt akıları etrafında yarım tur atar. AB olayındaki bir Φ akısı etrafında tam bir tur. Böylece kazanılan faz

$$\psi(\ldots,\vec{r}_i,\ldots,\vec{r}_j,\ldots) = e^{\overset{\text{anticlock}}{+}\,\overset{\text{Clockwise}}{-}\; i\frac{q\Phi}{\hbar c}}\,\psi(\ldots,\vec{r}_j,\ldots,\vec{r}_i,\ldots)$$

Eğer $q\Phi$ $0 - 2\pi$ aralığında sürekli değişirse iki parçacığın uyacağı istatistik BE ve FD arasında keyfi bir istatistik olur. Wilczek ne bozon ne de Fermiyon olan bu parçacığı "Anyon" olarak vaftiz etmiştir.

* Wu → Braid grup kavramını bulmuş. { FQHE, High T_c

NR yaklaşımında anyon yarım kuantlanmasını [illegible]

$(\vec{A}, \phi)$

$$L = \frac{1}{2} m \dot{\vec{r}}^{\,2} + \frac{q}{c}\vec{A}\cdot\dot{\vec{r}} - q\phi$$

<u>LD</u>

$\xrightarrow{\;v\;}$ x

$$A_x' = \frac{1}{\sqrt{1-\beta^2}}\left(A_x - \frac{v}{c}\phi\right)$$

$$A_y' = A_y$$

$$A_z' = A_z$$

$$\phi' = \frac{1}{\sqrt{1-\beta^2}}\left(\phi - \frac{v}{c}A_x\right)$$

BS K

X boyunca v hızı ile hareket eden statik bir MA kaynağı olsun. Bu çerçevede $(\vec{A}, 0)$ ← $\phi = 0$

LAB'da ($\beta = \frac{v}{c} \to 0$)

$$A_x' = \frac{1}{\sqrt{1-\beta^2}} A_x \simeq A_x \quad ; \quad A_y' = A_y \quad ; \quad A_z' = A_z$$

$$\phi' = \frac{1}{\sqrt{1-\beta^2}} \left(0 - \frac{(-v)}{c} A_x\right) = \frac{1}{c} \vec{v} \cdot \vec{A}$$

$$\vec{A}' = \vec{A} \qquad \phi' = \frac{\vec{v}}{c} \cdot \vec{A}$$

$\vec{v}$ hızı ile hareket eden statik MA kaynağı bu pot. üretecek.

bunu L de yerleştirirsek

$$L = \frac{1}{2} m \dot{\vec{r}}^2 + \frac{q}{c} \vec{A} \cdot (\dot{\vec{r}} - \vec{v})$$

Şimdi iki anyon sist.ine dönelim. Statik bir anyon civarında ne $\vec{E}$ ne de $\vec{B}$ var. Ama, mag. akı tüpü Φ nin varlığı etrafında (anyon'un merkezindeki) sıfırdan farklı bir vektör pot. jenere eder.

$$\vec{A}(\vec{r} - \vec{r}_i) = \frac{\Phi}{2\pi} \frac{\hat{\theta}}{|\vec{r} - \vec{r}_i|} , \quad \phi = 0$$ skaler pot.

$\vec{r}_i$: anyon merkezinin koord. vekt.

$\hat{\theta}$: $\vec{r} - \vec{r}_i$ ye dik olan açısal doğrultu boyunca birim vektör.

⟩ buna dayanarak, iki anyon sist.nin Lagranjiyenini yazabiliriz.

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$$L = \frac{1}{2} m \left(\dot{\vec r}_i^{\,2} + \dot{\vec r}_j^{\,2}\right) + \frac{q}{c} A(\vec r_i - \vec r_j)\cdot(\dot{\vec r}_i - \dot{\vec r}_j)$$

$$+ \frac{q}{c} A(\vec r_j - \vec r_i)\cdot(\dot{\vec r}_j - \dot{\vec r}_i)$$

$$= \frac{1}{2} m \left(\dot{\vec r}_i^{\,2} + \dot{\vec r}_j^{\,2}\right) + \frac{2q}{c} A(\vec r_i - \vec r_j)\cdot(\dot{\vec r}_i - \dot{\vec r}_j)$$

$A(\vec r_i - \vec r_j) = -A(\vec r_j - \vec r_i)$ kullanıldı

kanonik momentumlar:

$$\vec p_i = \frac{\partial L}{\partial \dot{\vec r}_i} = m\dot{\vec r}_i + \frac{2q}{c} A(\vec r_i - \vec r_j)$$

$$\vec p_j = \frac{\partial L}{\partial \dot{\vec r}_j} = m\dot{\vec r}_j + \frac{2q}{c} A(\vec r_j - \vec r_i)$$

$$H = \dot{\vec r}_i\cdot\vec p_i + \dot{\vec r}_j\cdot\vec p_j - L$$

$$= \frac{1}{2m}\left\{\left[\vec p_i - \frac{2q}{c} A(\vec r_i - \vec r_j)\right]^2 + \left[\vec p_j - \frac{2q}{c} A(\vec r_j - \vec r_i)\right]^2\right\}$$

N tane için genelleştirilirse

$$H = \sum_{i=1}^{N} \frac{1}{2m}\left[-\frac{\hbar}{i}\frac{\partial}{\partial \vec r_i} - \frac{2q}{c}\sum_{j\neq i}^{N} A(\vec r_i - \vec r_j)\right]^2$$

$$H\psi(\vec r_1, \dots, \vec r_N, t) = i\hbar\frac{\partial}{\partial t}\psi(\vec r_1, \dots, \vec r_N, t)$$

N anyon dalga fonksiyonu

$$\psi'(\vec r_1, \dots, \vec r_N, t) = \prod_{i<j} e^{-i\frac{q\Phi}{\hbar}\phi_{ij}}\,\psi(\vec r_1, \dots, \vec r_N, t)$$

$$\phi_{ij} = \arg(\vec r_i - \vec r_j) \Rightarrow H\psi' = i\hbar\frac{\partial\psi'}{\partial t} = \left(\sum_{i=1}^{N} \frac{-\hbar^2}{2m}\nabla_i^2\right)\psi'$$

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Anyon gazının 2. virial katsayısı:

$$H = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2}\right) + \frac{1}{2} m\omega^2 (x_1^2 + y_1^2 + x_2^2 + y_2^2)$$

$m = 1$

2D

$z_1 = x_1 + i y_1$

$z_2 = x_2 + i y_2$

$$Z = \frac{1}{2}(z_1 + z_2) \qquad z = z_1 - z_2$$

$$H_{CM} = -\frac{\partial^2}{\partial Z \partial Z^*} + \omega^2 Z Z^*$$

$$H_r = -4\frac{\partial^2}{\partial z \partial z^*} + \frac{1}{4}\omega^2 z z^*$$

$$H\Psi(Z,z) = (H_{CM} + H_r)\Psi(Z,z) = E\Psi(Z,z)$$

$E = E_{cm} + E_r$

#1 $H_{cm}\psi(Z) = E_{cm}\psi(Z) \qquad Z = R e^{i\alpha}$

$$\psi(Z) = \xi(R)\,\Theta(\alpha)$$

$$\Rightarrow \left(-\frac{1}{4}\frac{\partial^2}{\partial R^2} - \frac{1}{4R}\frac{\partial}{\partial R} - \frac{1}{4R^2}\frac{\partial^2}{\partial \alpha^2}\right)\xi\Theta + \omega^2 R\xi\Theta = E_c\xi\Theta$$

#2 $H_r \chi(z) = E_r \chi(z) \qquad z = r e^{i\phi}$

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$$\left(-\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}\right)\cancel{\chi}(z)+\frac{1}{4}\omega^2 r^2 \cancel{\chi}(z)=E_r\cancel{\chi}(z)$$

$\chi(z)$ değişkenlerini ayrıldığında tekil sınır koşulu ortaya çıkar.

$$\chi(z)=(z_1-z_2)^{\frac{\theta}{\pi}} f(z)=r^{\theta/\pi}\eta(r)\,e^{i\theta\frac{\phi}{\pi}}\Phi(\phi)$$

$$\equiv\tilde{\eta}(r)$$

Wilczek: $\theta=q\Phi$

$$\Phi(\phi)=e^{il\phi} \qquad l=0,\pm 2,\pm 4,\dots$$

$$\Phi(0)=\Phi(2\pi) \qquad \Phi(0)=\Phi(\pi)$$

$$-\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}(2\Delta+l)^2\right]\tilde{\eta}(r)+\frac{1}{4}\omega^2r^2\tilde{\eta}(r)=E_r\tilde{\eta}(r)$$

$$\Delta=\frac{\theta}{2\pi} \qquad (E_r-\omega)r^2=k^2r^2 \qquad \tilde{\eta}(r)=e^{-\frac{1}{4}\omega r^2}\eta(r)$$

$$\Downarrow$$

burada daha $|2\Delta+l|$ seçeceğiz

$$\frac{d^2\eta}{d(kr)^2}+\frac{1}{(kr)}\frac{d\eta}{d(kr)}+\left[1-\frac{(2\Delta+l)^2}{(kr)^2}\right]\eta=0$$

$$\Rightarrow \boxed{\eta(r)=J_{|2\Delta+l|}(kr)}$$

$\omega\to 0$ limiti olmadan çözülürse.

$$\tilde{\eta}=\frac{1}{\sqrt{r}}w=\frac{1}{\sqrt{r}}e^{-\frac{1}{2}\omega r^2}r^{(2\Delta+l+\frac{1}{2})}y(r)=r^{(2\Delta+l)}e^{-\frac{1}{4}\omega r^2}y(r)$$

$$\rho=\frac{1}{2}\omega r^2$$

$$\rho\frac{d^2y}{d\rho^2}+\left[|2\Delta+l|+1-\rho\right]\frac{dy}{d\rho}+\frac{1}{2}\left[\mathcal{E}-(|2\Delta+l|+1)\right]y=0$$

$$\mathcal{E}=E_r/\omega$$

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$$\rho L_n^{\alpha''}(\rho) + (\alpha+1-\rho) L_n^{\alpha'}(\rho) + n L_n^{\alpha}(\rho) = 0$$

$\alpha = |2\Delta + \ell|$ $\qquad \frac{1}{2}\left[\varepsilon - (|2\Delta+\ell|+1) \right] = n$ = tamsayılar $= 0, 1, 2, \ldots$

$$\Rightarrow E_r = (2n + |2\Delta + \ell| + 1)\,\omega$$

$$y(\rho) = L_n^{|2\Delta+\ell|}(\rho)$$

$$\Rightarrow \chi(z) = z^{|2\Delta+\ell|}\, e^{-\frac{1}{2}\omega z z^*}\, L_n^{|2\Delta+\ell|}\left(\frac{1}{2}\omega z z^*\right)$$

$(2\Delta+\ell)^2 \longrightarrow$ den dolayı $|2\Delta+\ell|$ seçtik.

benzer şekilde $\quad E_L = \omega\,[2N + |L| + 1]$

$$L = 0, \pm 1, \pm 2, \ldots$$

$$f(z) = Z^{|L|}\, L_N^{|L|}(2\omega Z Z^*)\, e^{-\omega Z Z^*}$$

$$p = nkT\,[1 + Bn + Cn^2 + \ldots]$$

$n = \frac{N}{V}$ top mol. sayı

$V = L^2$

2 nd, 3 rd

ideal Boltzman gazından ve hede sapmayı ölçer

$D = 3$ de $\quad B_0(T) = \frac{-\varepsilon \lambda^3}{2^{5/2}}$ $\quad \lambda = \sqrt{2\pi\hbar^2/mkT}$ idi

$$\varepsilon = \begin{cases} +1 & \text{BE} \\ -1 & \text{FD} \end{cases}$$

Benzer olarak $D = 2$ de $\quad B_0(T) = \frac{-\varepsilon\lambda^2}{4}$

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bare coll. eliminazione. $B(T) = \frac{1}{2} A - \frac{1}{2A} \lambda^4 Z_2$ + bozzetto 2 par. part. funct.

$$Z_2 = Z_2^{CM} \tilde{Z}_2 \quad [\ \bar{K} = \bar{k}_1 + \bar{k}_2 \quad \bar{R} = \frac{1}{2}(\bar{r}_1 + \bar{r}_2)\ ]$$

$$Z_2^{CM} = \mathrm{Tr}\ e^{-\beta H_0^{CM}} = \frac{2A}{\lambda^2}$$

$$\tilde{Z}_2 : \quad [\ k = \frac{1}{2}(k_1 - k_2) \quad \bar{r} = \bar{r}_1 - \bar{r}_2\ ]$$

$$\tilde{Z}_2 = \mathrm{Tr}\ e^{-\beta H_r} = \int dr\ \langle \bar{r} | e^{-\beta H_r} | \bar{r} \rangle$$

$$= \frac{V^2}{(2\pi)^4} \int d\bar{r} \int d\bar{k} \int d\bar{k}' \langle \bar{r} | \bar{k} \rangle \langle \bar{k} | e^{-\beta \hbar^2 k^2 / m} | \bar{k}' \rangle \langle \bar{k}' | \bar{r} \rangle$$

$$\langle \bar{r} | \bar{k} \rangle = \frac{1}{\sqrt{V}} e^{i \bar{k} \cdot \bar{r}} \qquad \text{Boltzmann}$$

$$\langle \bar{r} | \bar{k} \rangle = \sqrt{\frac{2}{V}} \cos \bar{k} \bar{r} \qquad \text{BE}$$

$$\langle \bar{r} | \bar{k} \rangle = \sqrt{\frac{2}{V}} \sin \bar{k} \cdot \bar{r} \qquad \text{FD}$$

$$B_0(T)\Big|_{\text{Boltz}} = 0$$

$$B_0(T)\Big|_{BE} = -\frac{1}{4} \lambda^2$$

$$B_0(T)\Big|_{FD} = \frac{1}{4} \lambda^2$$

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$\star 1$ ya da $\star 2$ Boltzman gazı ile karşılaştırıldığında BE (ya da FD tipi) tipli quantum korelasyon parçacıkları arası çekici (itici) bir çeşit etkileşme hesaba katılır gibi.

$$B(T),\ Z_2^{(n)},\ \tilde{Z}_2 \Rightarrow B(T) - B_0(T) = \lambda^2 \left[\tilde{Z}_2(0) - \tilde{Z}_2(\Delta) \right]$$

$$\Delta = 0 \Rightarrow \text{BE} \qquad B_0(T) = -\frac{1}{4}\lambda^2 \qquad 0 \leq \Delta \leq 1$$

önemli nokta $\tilde{Z}_2(\Delta)$ hesabında yatar.

Anyonları önceliğe bir harmonik osilatör pot. içinde hapsolmuş gibi düşünebiliriz ve sonra pot.'i bir anda kaldırabiliriz $\omega \to 0$.

$$\tilde{Z}_2(\Delta) = \operatorname{Tr} e^{-\beta H_r} = \lim_{\omega \to 0} \operatorname{Tr} e^{-\beta \omega (2n + |2\Delta + l| + 1)}$$

$$B(T) - B_0(T) = 2\lambda^2 \lim_{\omega \to 0} \sum_{\substack{n = 0,1,2,\ldots \\ l = 0, \pm 2, \ldots}} \left\{ e^{-\beta\omega(2n + |l| + 1)} - e^{-\beta\omega(2n + |2\Delta + l| + 1)} \right\}$$

değerlendirilir

$$0 \leq \Delta \leq 1 \qquad |2\Delta + l| = 2\Delta + l \qquad l = 0, 2, \ldots$$
$$= |l| - 2\Delta \qquad l = -2, -4, \ldots$$

$$= \lim_{\omega \to 0} 2\lambda^2 \frac{e^{-\beta\omega}}{1 - e^{-2\beta\omega}} \left\{ \frac{1 - e^{-2\beta\omega\Delta}}{1 - e^{-2\beta\omega}} + \frac{e^{-2\beta\omega}(1 - e^{-2\beta\omega\Delta})}{1 - e^{-2\beta\omega}} \right\}$$

$$= 2\lambda^2 \lim_{\omega \to 0} \frac{\operatorname{sh} \beta\omega\Delta \ \operatorname{sh}[\beta\omega(1-\Delta)]}{\operatorname{sh}^2(\beta\omega)} = 2\lambda^2 \Delta(1-\Delta)$$

$$\frac{4}{\lambda^2} B(T) = 8\Delta(1-\Delta) - 1$$

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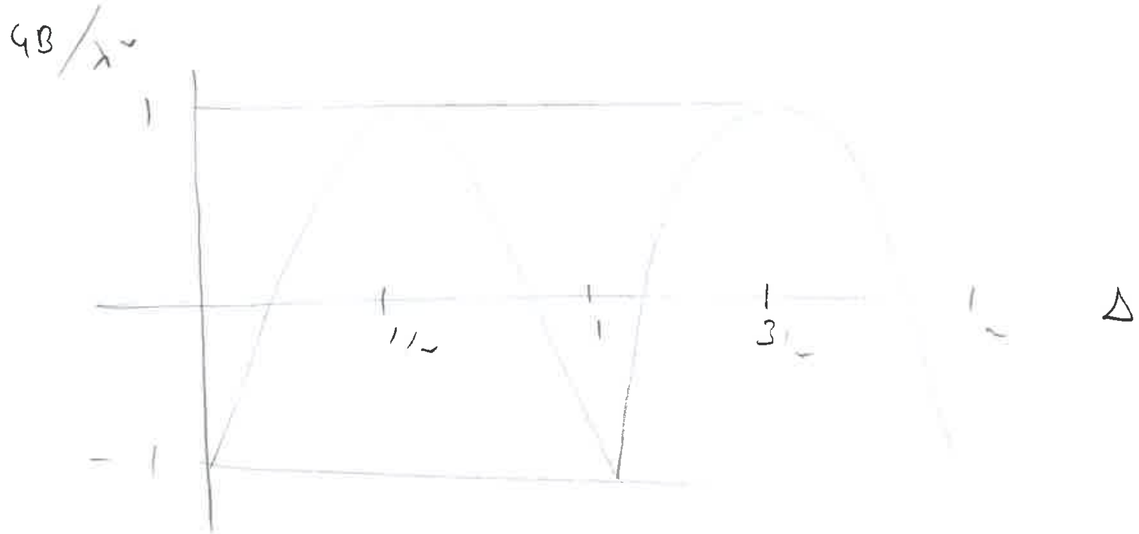

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PRL 48, 1144 (1982)

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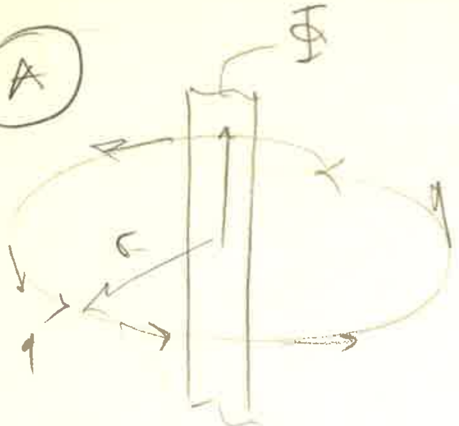

$$\mathcal{E} = -\frac{d\Phi}{dt} \qquad \oint \vec{E}\cdot d\vec{s}$$

bir tur için yapılan iş

$$W = q\,\mathcal{E} = q\,E(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$

$$E = -\frac{1}{2\pi r}\frac{d\Phi}{dt} = -\frac{1}{2\pi r}\dot{\Phi}$$

$$\vec{E} \nearrow\nearrow \hat{z}\times\hat{r} = \frac{1}{r}\hat{z}\times\vec{r}$$

$$\boxed{\vec{E} = \frac{-1}{2\pi r^2}\hat{z}\times\vec{r}\,\dot{\Phi}} = \frac{-1}{2\pi[x^2+y^2]}\hat{z}\times\vec{r}\,\dot{\Phi}$$

$$\frac{dL_z}{dt} = N_z = [\vec{r}\times(q\vec{E})]_z$$

$$= \left\{\vec{r}\times\left[\frac{q}{2\pi r^2}\hat{z}\times\vec{r}\,\dot{\Phi}\right]\right\}_z$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

$$= \frac{-q}{2\pi r^2}\left\{\vec{r}\times(\hat{z}\times\vec{r})\right\}_z\dot{\Phi}$$

$$= \frac{-q}{2\pi r^2}\left\{\vec{r}^{\,2}\hat{z} - \cancel{(\vec{r}\cdot\hat{z})}^{\,0}\vec{r}\right\}_z\dot{\Phi}$$

$$= -\frac{q}{2\pi}\dot{\Phi} \qquad \boxed{\Delta L_z = -q/2\pi\,\Delta\Phi} \Leftarrow \frac{\Delta L_z}{\Delta t} = -\frac{q}{2\pi}\frac{\Delta\Phi}{\Delta t}$$

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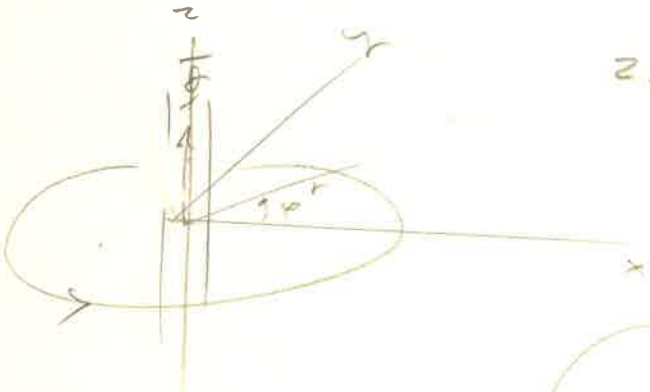
$$\Delta L_z = -q\,\Phi/2\pi$$

karşılık gelen kuantumlanmış açısal mom.,

$$L_z = \text{tam sayı} - \frac{q\Phi}{2\pi}$$

(B) Aynı sonuca şu şekilde de ulaşabiliriz.



z etrafındaki dönmeler

$$L_z = -i\frac{\partial}{\partial\varphi} - qA_\varphi$$

ile jenere edilir.

$$A_\varphi = \frac{\Phi}{2\pi}$$

$$\psi_n \propto e^{in\varphi}$$

$$L_z\psi_n = \left(n - \frac{q\Phi}{2\pi}\right)\psi_n$$

Bir evvelki ile uyumlu!

(C)

bu pot. bir singüler ayar ile elimine edilebilir.

$$\vec{A} \to \vec{A} - \vec{\nabla}\Lambda \qquad \Lambda = \frac{\Phi\varphi}{2\pi}$$

Singüler çünkü φ çokdeğerli fonk.

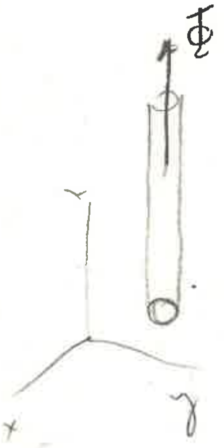
$$\psi'(\varphi+2\pi) = e^{-iq\Phi}\psi'(\varphi) \qquad \psi'(\varphi) = e^{i\left(\text{tam sayı} - \frac{q\Phi}{2\pi}\right)\varphi}$$

BS K

PRL, F. Wilczek, 48, 1144 (1982)

PR. 115 Aharonov & Bohm, 485 (1959)

PRD 12, 3845 (1975)



$$A_\varphi = \Phi/2\pi r \qquad A_r = 0 \qquad A_z = 0$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{r}\begin{vmatrix} \bar{r} & r\bar{\varphi} & \bar{z} \\ \partial_r & \partial_\varphi & \partial_z \\ A_r & rA_\varphi & A_z \end{vmatrix}$$

$$= \frac{1}{r}\cdot\begin{vmatrix} \bar{r} & r\bar{\varphi} & \bar{z} \\ \partial_r & \partial_\varphi & \partial_z \\ 0 & \Phi/2\pi & 0 \end{vmatrix} = 0 = \vec{B}$$

A_φ can be eliminated by a gauge transformation

$$A_i' = A_i - \partial_i \Lambda$$

However, the Hamiltonian is changed to

$$\frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2 \to \frac{1}{2\mu}\left(\frac{\hbar}{i}\vec{\nabla} + \frac{e}{c}\vec{A} - \frac{e}{c}\vec{\nabla}\Lambda\right)^2$$

To save the gauge invariance we transform

$$\psi \to e^{i\Lambda}\psi$$

$$\frac{1}{2\mu}\left(\frac{\hbar}{i}\vec\nabla + \frac{e}{c}\vec A - \frac{e}{c}\vec\nabla\Lambda\right)\cdot\left(\frac{\hbar}{i}\vec\nabla + \frac{e}{c}\vec A - \frac{e}{c}\vec\nabla\Lambda\right) e^{i\Lambda}\psi$$

$$= \frac{1}{2\mu}\left(\frac{\hbar}{i}\vec\nabla + \frac{e}{c}\vec A - \frac{e}{c}\vec\nabla\Lambda\right)\cdot\left[e^{i\Lambda}\left(\frac{\hbar}{i}\vec\nabla\psi + \hbar\vec\nabla\Lambda\psi + \frac{e}{c}\vec A\psi - \frac{e}{c}\vec\nabla\Lambda\psi\right)\right]$$

$$= \frac{1}{2\mu} e^{i\Lambda}\left(\frac{\hbar}{i}\vec\nabla + \frac{e}{c}\vec A - \frac{e}{c}\vec\nabla\Lambda + \hbar\vec\nabla\Lambda\right)^2\psi$$

$$\Lambda = +\frac{e}{\hbar c} f \qquad A'_\varphi = \cancel{A_\varphi - \frac{1}{r}\frac{\partial}{\partial\varphi}\frac{\Phi\varphi}{2\pi}} = 0$$

$$f = \frac{\hbar c}{e}\frac{\varphi}{2\pi}\Phi$$

$$\boxed{\Lambda = \Phi\varphi/2\pi}$$

$$\psi'(\varphi) = e^{iq\Phi\varphi/2\pi}\psi(\varphi)$$

$$\psi'(\varphi+2\pi) = e^{iq\Phi}\psi'(\varphi)$$

$$\psi'(\varphi) \sim e^{im\varphi}$$

$$\Rightarrow \quad m = \text{integer} + \frac{q\Phi}{2\pi}$$ spectrum of orbital angular momenta

In the prime syst. charged particle obeys free Schrödinger Eq.

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$\Psi \equiv \Psi(R,\theta;r,\varphi)$

$$\Psi(R,\theta;r,\varphi+2\pi) = e^{4\pi i\Delta}\,\Psi(R,\theta;r,\varphi)$$

$\Delta \equiv q\Phi/2\pi$

$(n+m+1)\hbar\omega$

$H = \dfrac{P^2}{2\mu} + \dfrac{1}{2}\mu\omega^2 r^2$ 2D Harmonic well.

$\cancel{E_{nm} = (2n+|m|+1)\hbar\omega}$

$\hbar\omega$

Δ

$n=0$	$m=0$	$E = \epsilon$	— 1
$n=1$	$m=0$	$= 2\epsilon$	— 2
$n=0$	$m=1$	$= 2\epsilon$	
$n=2$	$m=0$	3ϵ	— 3
$n=1$	$m=1$	3ϵ	
$n=0$	$m=2$	3ϵ	

$(\epsilon, 1)$ $(2\epsilon, 2)$ $(3\epsilon, 3)$, ...

COM

$(\epsilon,1)$ $(2\epsilon,2)$ $(3\epsilon,3)$, . . .

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$$E_n = (n+1)\hbar\omega \qquad n = 2n_r + |m|$$

$$= (2n_r + |m| + 1)\hbar\omega$$

			E	D
$n_r = 0$	$m = 0$	$2n_r + \vert m\vert = 0$	ϵ	1
$n_r = 0$	$m = \pm 1$	$2n_r + \vert m\vert = 1$	2ϵ	2
$n_r = 0$ $n_r = 1$	$m = \pm 2$ $m = 0$	$2n_r + \vert m\vert = 2$	3ϵ	3
$n_r = 0$ $n_r = 1$ $n_r = 2$	$m = \pm 3$ $m = \pm 1$ $m = 0$	$2n_r + \vert m\vert = 3$	4ϵ	4

tek uygun

2D Harmonic well.

K.M. için de aynı

$$F = (2n_r + |2j + 2\Delta| + 1)\hbar\omega \qquad j = 0, \pm 1, \pm 2, \pm 3, \ldots$$

BS K

		E/ϵ	D
$n_r = 0$	$j = 0$	$1 + 2\Delta$	1
$n_r = 0$	$j = 1$	$3 + 2\Delta$	2
$n_r = 1$	$j = 0$	$3 + 2\Delta$	
$n_r = 0$	$j = 2$	$5 + 2\Delta$	3
$n_r = 1$	$j = 1$	$5 + 2\Delta$	
$n_r = 2$	$j = 0$	$5 + 2\Delta$	

⋮

$n_r = 0$	$j = -1$	$3 - 2\Delta$	1
$n_r = 0$	$j = -2$	$5 - 2\Delta$	2
$n_r = 1$	$j = -1$	$5 - 2\Delta$	
$n_r = 0$	$j = -3$	$7 - 2\Delta$	3
$n_r = 1$	$j = -2$	$7 - 2\Delta$	
$n_r = 2$	$j = -1$	$7 - 2\Delta$	

$$Z = \frac{1}{2}\left[(x_1 + x_2) + i(y_1 + y_2) \right]$$

$$Z^* = \frac{1}{2}\left[(x_1 + x_2) - i(y_1 + y_2) \right]$$

$$ZZ^* = \frac{1}{4}\left[(x_1 + x_2)^2 + (y_1 + y_2)^2 \right]$$

$$= \frac{1}{4}\left(x_1^2 + y_1^2 + x_2^2 + y_2^2 + 2x_1x_2 + 2y_1y_2 \right)$$

$$z = z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z^* = z_1^* - z_2^* = (x_1 - x_2) - i(y_1 - y_2)$$

$$zz^* = (x_1 - x_2)^2 + (y_1 - y_2)^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2$$

$$- 2x_1x_2 - 2y_1y_2$$

$$4ZZ^* + zz^* = 2\left(\underline{x_1^2 + y_1^2 + x_2^2 + y_2^2} \right)$$

$$x_1^2 + y_1^2 + x_2^2 + y_2^2 = 2ZZ^* + \frac{1}{2} zz^*$$

$$\frac{\partial}{\partial x_1} = \frac{\partial Z}{\partial x_1}\frac{\partial}{\partial Z} + \frac{\partial z}{\partial x_1}\frac{\partial}{\partial z}$$

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